STAT 702/J702

October 12th, 2004

-Lecture 16-

Instructor: Brian Habing
Department of Statistics
Telephone: 803-777-3578
E-mail: habing@stat.sc.edu

Today

- Homework Solutions
- The Bivariate Normal
- Functions of Joint Random Variables

Ch 3 #1) p(x,y) is:

y/x	1	2	3	4
1	.10	.05	.02	.02
2	.05	.20	.05	.02
3	.02	.05	.20	.04
4	.02	.02	.04	.10

a) Find the marginal pmf.'s for X & Y

b) Find $p_{X|Y}(x|1)$ and $p_{Y|X}(y|1)$

Ch. 3 #8)

$f_{XY}(x,y) = (6/7)\ (x+y)^2 \quad 0 \le x,y \le 1$

a) Find the marginal densities of X & Y.

b) Find the two conditional densities.

Example 3)

$$f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_x)^2}{\sigma_x^2} - \frac{2\rho(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y} + \frac{(y-\mu_y)^2}{\sigma_y^2}\right]\right\},$$

$-\infty<\mu_x,\ \mu_y<\infty,\quad \sigma_x,\ \sigma_y > 0,\ \ -1< \rho <1$

$$F(x_0, y_0) = \int_{-\infty}^{x_0}\int_{-\infty}^{y_0} f(x,y)\,dx\,dy$$

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)dy$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)dx$$

$$f_{Y|X}(y\,|\,x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

$$f_{X|Y}(x\,|\,y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

3.6.2 – Functions of Joint Random Variables

(X, Y) have joint pdf $f_{XY}(x, y)$.

We want the distribution of $U=g_1(X, Y)$, $V=g_2(X, Y)$.

E.g. : U=X+Y, $V=Y^2$ or U=X+Y, V=X-2Y, etc.

For the continuous case the joint pdf of (U,V) is

$f_{U,V}(u, v) = f_{X,Y}(h_1(u, v), h_2(u, v))\ |J|$

where h_1 and h_2 are the inverse functions: $x = h_1(u, v)$, $y = h_2(u, v)$)

And J is the Jacobian $J = \begin{vmatrix} \frac{dh_1}{du} & \frac{dh_1}{dv} \\ \frac{dh_2}{du} & \frac{dh_2}{dv} \end{vmatrix}$

Example 1) X and Y have joint p.d.f.
$f_{XY}(x,y) = 2 \quad 0 \leq x,y \leq 1$

U=X/Y and V=Y

Find the joint and marginal p.d.f's of X and Y.

Example 2) X and Y are bivariate normal with means 0, variances 1, and correlation = 0.

Let $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Find the joint and marginal distributions.