STAT 702/J702

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-Lecture 14-

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Today

- Homework
- Joint Distributions (continued)

Chapter 2 #67: The Weibull has CDF

$$F(x) = 1 - e^{-(x/\alpha)^{\beta}}$$

$$\text{for } \; x \geq 0, \; \alpha > 0, \; \text{and } \; \beta > 0$$

a) Find the density function.

b) Show if W follows a Weibull, then $X=(W/\alpha)^\beta$ follows an exponential.

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c) How could Weibull random variables be generated from a uniform random generator?

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Also) Use R to plot the pdf for a few values of alpha and beta to demonstrate how they affect the behavior of the Weibull distribution.

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Chapter 3 – Joint Distributions

The joint behavior of two random variables X and Y is determined by there CDF:

$$F_{XY}(x, y) = P(X \le x, Y \le y)$$

3.2 - Discrete R.V.'s

For discrete R.V.'s the joint p.m.f. is

$$p(x, y) = P(X=x, Y=y)$$

Example) A fair coin is tossed three times. Let X=number of heads in three tossings and Y= difference (in absolute values) between the number of heads and number of tails.

The Marginal p.m.f of X is

$p_X(x) = \Sigma_y\, p(x, y)$

The Conditional p.m.f. of X is

$p_{X|Y}(x \mid y) = P(X=x \mid Y=y)$

$= P(X=x, Y=y) / P(Y=y)$

$= p_{XY}(x, y)/p_Y(y)$

X and Y are independent if

$F_{XY}(x, y) = F_X(x)\, F_X(y)$

This implies that

$p_{XY}(x, y) = p_X(x)\, p_Y(y)$

It also works for functions g(x) and h(y).

3.3 Continuous R.V.'s

Continuous (X, Y) have joint cdf

$F_{XY}(x,y) = P(X \leq x, Y \leq y)$

The joint pdf is

$$f_{XY}(x, y) = \frac{\partial^2}{\partial x\, \partial y} F_{XY}(x, y)$$

So

$$F(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f(u,v)dudv$$

and

$$P((x,y) \in A) = \iint_A f(x,y)dxdy$$

The marginal p.d.f.'s are analogous to the marginal p.m.f.'s for discrete variables, but are defined using integrals:

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)dy, \qquad f_Y(y) = \int_{-\infty}^{\infty} f(x,y)dx.$$

The conditional p.d.f.'s are also analogous:

$$f_{Y|X}(y \mid x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

Example 1) $f(x, y) = 2,\ 0 < x < y < 1$
and 0 elsewhere

Example 2)

$f(x, y) = \lambda^2 \exp(-y\lambda)$
$0 \le x \le y \le 1,\ \lambda > 0$

Example 3)

$$f(x, y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_x)^2}{\sigma_x^2} - \frac{2\rho(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y} + \frac{(y-\mu_y)^2}{\sigma_y^2}\right]\right\},$$

$-\infty < \mu_x, \mu_y < \infty, \quad \sigma_x, \sigma_y > 0, \ -1 < \rho < 1$