STAT 702/J702

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-Lecture 13-

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Today

- Functions of Continuous Distributions (continued)
- Joint Distributions

2.2.2-The Gamma Distribution

$$g(t) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)} t^{\alpha-1} e^{-\lambda t} \text{ for } t \geq 0$$

2.2.3 – The Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \text{ for } -\infty < x < \infty$$

2.3 - Functions of Random Variables

Let Y=aX+b

$F_Y(y) = F_X((y-b)/a)$

$f_Y(y) = (1/a)\, f_X((y-b)/a)$

e.g. if X~N(μ, σ^2) then (X- μ)/ σ~Z

Y=g(X): Let X be a continuous RV with p.d.f. $f(x)$ and Y=g(X), where g is differentiable and strictly monotone every where that $f(x)$>0.

Then

$$f_Y(y) = f_X(g^{-1}(y))\left|\frac{d}{dy}g^{-1}(y)\right|$$

Page 60: "For any specific problem, it is usually easier to proceed from scratch than to decipher the notation and apply the proposition."

Example) Let $X=Z^2$ where $Z \sim N(0,1)$

Example 2) Let $X=F^{-1}(U)$ where U is uniform on [0,1] and F is a CDF.

Chapter 3 – Joint Distributions

The joint behavior of two random variables X and Y is determined by there CDF:

$$F_{XY}(x, y) = P(X \leq x, Y \leq y)$$

We can use this definition to find the area of any given rectangle:

$P(x_1 < X \leq x_2, y_1 < Y \leq y_2) =$
$F_{XY}(x_2, y_2) - F_{XY}(x_1, y_2)$
$- F_{XY}(x_2, y_1) + F_{XY}(x_1, y_1)$,
for $x_1 < x_2$, $y_1 < y_2$.

3.2 - Discrete R.V.'s

For discrete R.V.'s the joint p.m.f. is

$$p(x, y) = P(X=x, Y=y)$$

Example) A fair coin is tossed three times. Let X=number of heads in three tossings and Y= difference (in absolute values) between the number of heads and number of tails.