STAT 702/J702

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-Lecture 12-

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Today

- Exam Solutions
- Functions of Continuous Distributions

1) Consider the random variable X defined by the p.m.f.

x	-1	0	1	2	5
p(x)	0.5	0.1	0.1	0.25	0.05

Graph the c.d.f. for X and find its mean and standard deviation.

2) A gourmet pizza place offers a special where you can choose 2 of their 5 cheeses and 4 of their 20 vegetables. How many different pizzas are possible with this special?

3) Machines 1 and 2 each produce 20,000 parts per day with a defective rate of 1.0%. Machines 3, 4, and 5 each produce 15,000 parts per day with a defective rate of 1.3%. What is the probability that a randomly selected defective was produced by machine 1?

4) Consider events A and B with P(A)=0.2 and P(B)=0.2. Can A and B be both disjoint and independent? Justify your answer mathematically using the definitions.

5) The brother has probability p_1 of making each shot he attempts and the sister has probability p_2 of making each shot she attempts. The first one to make it when the other misses is the winner. You may assume the shots are ind. of each other. Let X=# rounds the game takes to finish. Name the distribution of X, give the value of its parameter(s), and the expected number of rounds it will take for the game to finish.

6) Consider taking a sample of size n from a population of size N with percentage of "successes"= p. One of the issues in deciding whether to use the binomial to approximate the hypergeometric is how different the estimated standard deviation will be. What is the largest (in terms of N and/or p) that the sample size n be so that $1 \geq \sigma_{hyper}/\sigma_{binomial} \geq 0.95$.

7) Potholes occur at a rate of approximately 5 per mile on a stretch of highway. Choose an appropriate distribution to assume and estimate the probability that no potholes will be encountered in the next ½ mile?

8) An in-depth survey of the U.S. senate managed to contact 45 randomly chosen senators out of the 100. If only 5% of all 100 senators agree with the position we are investigating, what is the probability that at least one of those in our sample agreed with the position?

9) An assembly line produces parts with a defective rate of 1.5%. Choose an appropriate distribution to assume and estimate the number of defectives that will be found in the next 20 that are sampled.

10) Consider a geometric random variable X with parameter p. Find the values of p that minimize and maximize Var(X) and the corresponding values of the variances. Make sure to justify your answer, including using calculus to find any potential local minima and maxima.

11) Consider a negative binomial random variable X with parameters p and r. Use the ratio of consecutive terms (and simplify the resulting expression) to give the formula for finding the value of k that maximizes P[X=k].

2.2.2-The Gamma Distribution

$$g(t) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)} t^{\alpha-1} e^{-\lambda t} \quad \text{for } t \geq 0$$

2.2.3 – The Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \quad \text{for } -\infty < x < \infty$$

2.3 - Functions of Random Variables

Let Y=aX+b

$F_Y(y) = P(Y \leq y)$

$= P(aX+b \leq y)$

$= P(X \leq (y-b)/a)$

$= F_X((y-b)/a)$

$f_Y(y) = dF_Y(y)$

$= dF_X((y-b)/a)$

$= (1/a)\, f_X((y-b)/a)$

Say X is Normal (μ, σ^2) …

<u>Y=*g* (X):</u> Let X be a continuous RV with p.d.f. $f(x)$ and Y=g(X), where g is differentiable and strictly monotone every where that $f(x)>0$.

Then

$$f_Y(y) = f_X(g^{-1}(y))\left|\frac{d}{dy}g^{-1}(y)\right|$$

Page 60: "For any specific problem, it is usually easier to proceed from scratch than to decipher the notation and apply the proposition."