STAT 702/J702

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-Lecture 11-

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Today

- Homework Solutions
- Gamma Distribution
- Normal Distribution

Ch. 2 #31b) Phone calls are received at a certain residence as a Poisson process with parameter λ=2 per hour.

Ch. 2 #40) Suppose that X has the density function $f(x)=cx^2$ for $0 \le x \le 1$ and 0 otherwise.

a) Find c.

b) Find the cdf.

c) What is $P(0.1 \le X < 0.5)$?

2.2.1 – Exponential Distribution

In a Poisson process let
X=time until the next occurrence

$f(x) = \lambda e^{-\lambda x}$

$E(X) = 1/\lambda$

$Var(X) = 1/\lambda^2$

Example: A high speed network suffers brief spot outages according to a Poisson process with rate of 0.001 per hour.

a) What is the probability that the next outage will happen within the coming hour?

b) What is the expected amount of time until the next outage?

c) How many outages do you expect to occur in the next 72 hours?

d) A failure occurs at 2.46 hours. What is the probability that the next failure occurs between 2.46 and 3.46 hours?

2.2.2-The Gamma Distribution

$$g(t) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)} t^{\alpha-1} e^{-\lambda t} \text{ for } t \geq 0$$

Where $\Gamma(x) = \int_0^{\infty} u^{x-1} e^{-u} du$ for $x > 0$

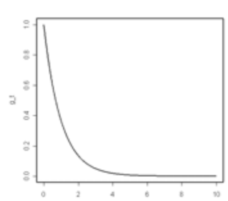
α is called the shape parameter and λ is called the scale parameter

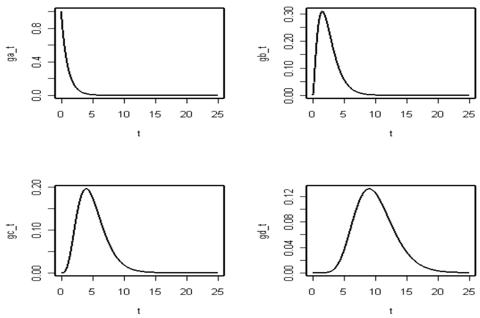
We will see later that:

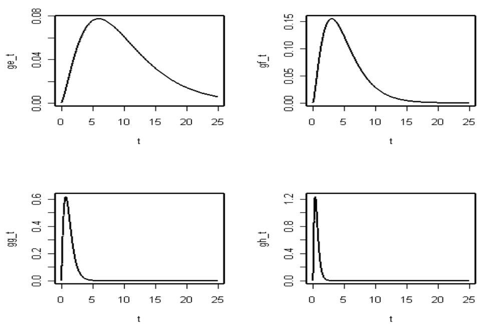
$E(T)=\alpha/\lambda$

$Var(T)=\alpha/\lambda^2$


```
t<-(0:1000)/100
g_t<-dgamma(t,shape=1,rate=1)
plot(t,g_t,xlim=c(0,10),
  type="l")
```





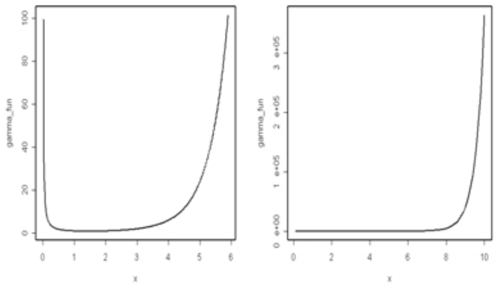


The Gamma Function (Ch.2 #49)

$$\Gamma(x) = \int_0^\infty u^{x-1} e^{-u} du \text{ for } x > 0$$

a) $\Gamma(1)=1$ b) $\Gamma(x+1)=x\,\Gamma(x)$

c) $\Gamma(n)=(n-1)!$ d) $\Gamma(\tfrac{1}{2}) = \text{sqrt}(\pi)$



2.2.3 - The Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \text{ for } -\infty < x < \infty$$

2.3 - Functions of Random Variables

Let Y=aX+b

$F_Y(y) = P(Y \le y)$

$= P(aX+b \le y)$

$= P(X \le (y-b)/a)$

$= F_X((y-b)/a)$

$f_Y(y) = dF_Y(y)$

$= dF_X((y-b)/a)$

$= (1/a)\, f_X((y-b)/a)$

Say Y is Normal (μ, σ^2) …