STAT 702/J702

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-Lecture 9-

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Today

- Homework Solutions
- Poisson Distribution
- Continuous Random Variables

Ch 4 #45) $E(X)=E(Y)=\mu$, but $\sigma_x \neq \sigma_y$.
Let $Z=\alpha X+(1-\alpha)Y$.

a) Show that $E(Z)=\mu$

b) Find α in terms of σ_x and σ_y to minimize Var(Z).

c) Under what circumstances is it better to use the average (X+Y)/2 than either X or Y alone?

2.1.5 - Poisson Process

A Poisson process with parameter λ is a model for events that occur over time (or space, etc…) such that:

1. Occurrences of events in non-overlapping intervals are independent.
2. The probability of exactly one change in an interval of length h is $\lambda h + o(h)$.
3. The probability of two or more occurences in an interval of length h is $o(h)$.

Examples include:

- The number of radioactive particles emitted by a radioactive isotope.
- Number of people arriving in a line.
- The number of phone calls arriving at a telephone exchange.

A Poisson process is very similar to a binomial experiment where the small sub-intervals constitute the trials and X is the number of occurrences.

In fact we can derive the p.d.f. of the Poisson distribution by taking a binomial and letting $n \to \infty$ and $np \to \lambda$.

For the Poisson Distribution we have:

$$P(X = x) = \frac{\lambda^x}{x!} e^{-\lambda}$$

$$\mu_X = \lambda$$

$$\sigma_X^2 = \lambda$$

Example: Between 2am and 4am cars pass the mile marker at a rate of 24 per hour. What is the probability that 0 cars will pass in a 5 minute span?

One car?

What is the expected number of cars to pass by in the 5 minute span?

How long until the next car passes?

2.2 – Continuous Variables

Consider a random number generator that selects a real number at random from between 0 and 1.

The probability density function (pdf) $f(x)$ satisfies the following:

a) $f(x) \geq 0$ for all x.

b) $\int_{-\infty}^{+\infty} f(x)dx = 1$

c) $P(a < X < b) = \int_{a}^{b} f(x)dx$

The cdf is defined the same way:

F(X) = P(X≤x) = $\int_{-\infty}^{x} f(x)dx$

For expected values we need to change the summation into an integral:

$$E(X) = \sum xp(x) \quad \Rightarrow \quad \int_{-\infty}^{+\infty} xf(x)dx$$

$$Var(X) = \sum (x-\mu)^2 p(x)$$

$$\Rightarrow \quad \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)dx$$

Three notes:

- $f(x)=F'(x)$
- $P(a<X<b)=F(b)-F(a)$
- $P(x<X<x+dx)\approx f(x)dx$