STAT 702/J702

September 14th, 2004

-Lecture 8-

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Today

- Homework Solutions
- μ and σ^2 of the Hypergeometric
- Geometric Distribution
- Negative Binomial Distribution

Ch. 2 #1) Suppose X is a discrete random variable with P(X=0)=0.25, P(X=1)=0.125, P(X=2)=0.125, and P(X=3)=0.5. Graph the p.m.f. and c.d.f. and calculate the mean and variance.

Ch. 2 #11) Consider the binomial distribution with n trials and probability p of success on each. For what value of k is P(X=k) maximized?

Hint: Similar strategy to example I on page 13.

Binomial Distribution

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x} \text{ for } x \in 0,\ldots n$$

$$\mu_X = np$$

$$\sigma_X^2 = np(1-p)$$

$$\sigma_X = \sqrt{np(1-p)}$$

Hypergeometric Distribution

$$p(x) = \frac{\binom{r}{x}\binom{n-r}{m-x}}{\binom{n}{m}} \text{ for } x \in 0,\ldots \min(m,r)$$

Finding the mean and variance takes some algebra…

$$\mu_X = \sum_{x=0}^{\min(m,r)} x \frac{\binom{r}{x}\binom{n-r}{m-x}}{\binom{n}{m}}$$

Now notice that $\binom{n}{m} = \frac{n}{m}\binom{n-1}{m-1}$

So

$$\mu_X = \sum_{x=0}^{\min(m,r)} x \frac{\frac{r!}{x!(r-x)!}\binom{n-r}{m-x}}{\frac{n}{m}\binom{n-1}{m-1}}$$

Now also notice that $x/x!=1/(x-1)!$ when $x\neq0$. Also, we can factor out the n and m as constants relative to the sum.

$$\mu_X = \left(\frac{m}{n}\right) \sum_{x=0}^{\min(m,r)} \frac{\frac{r!}{(x-1)!(r-x)!}\binom{n-r}{m-x}}{\binom{n-1}{m-1}}$$

$$= \left(\frac{m}{n}\right) \sum_{x=0}^{\min(m,r)} \frac{\left(\frac{r(r-1)!}{(x-1)!(r-x)!}\right)\binom{n-r}{(m-1)-(x-1)}}{\binom{n-1}{m-1}}$$

$$\mu_X = \left(r\frac{m}{n}\right)\sum_{x=0}^{\min(m,r)} \frac{\left(\frac{(r-1)!}{(x-1)!(r-x)!}\right)\binom{n-r}{(m-1)-(x-1)}}{\binom{n-1}{m-1}}$$

$$= \left(r\frac{m}{n}\right)\sum_{x=0}^{\min(m,r)} \frac{\binom{r-1}{x-1}\binom{n-r}{(m-1)-(x-1)}}{\binom{n-1}{m-1}} = \left(r\frac{m}{n}\right)$$

Using the fact that

$$\binom{n}{m} = \frac{n(n-1)}{m(m-1)}\binom{n-2}{m-2}$$

We can also get that

$$\sigma_X^2 = m\left(\frac{r}{n}\right)\left(\frac{n-r}{n}\right)\left(\frac{n-m}{n-1}\right)$$

If we use n for sample size, N for population size, and say $p=r/n$:

$$\mu_X = m\left(\frac{r}{n}\right) \Rightarrow np$$

$$\sigma_X^2 = m\left(\frac{r}{n}\right)\left(\frac{n-r}{n}\right)\left(\frac{n-m}{n-1}\right)$$

$$\Rightarrow np(1-p)\frac{(N-n)}{(N-1)}$$

Geometric Distribution

Consider a binomial experiment where you continue sampling until you encounter the first "success".

Let X=# of the trial where the 1st success is observed.

Note that this is a proper distribution as:

$$\sum_{x=1}^{\infty}(1-p)^{x-1}p \quad = \sum_{x=0}^{\infty}(1-p)^{x}p$$

$$= p\sum_{x=0}^{\infty}(1-p)^{x} \; = p\left(\frac{1}{1-(1-p)}\right)$$

$$= \frac{p}{p} = 1$$

To calculate the expected value and variance we can use a calculus trick.

Example: What is the probability that coin flipping doesn't result in a head until the fifth flip?

Negative Binomial Distribution

Let X=# of the trial where the r^{th} success is observed.

The negative binomial has

$$\mu_X = r\left(\frac{1}{p}\right) \text{ and } \sigma_X^2 = \frac{r(1-p)}{p^2}$$