STAT 702/J702
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Today

- Homework Solutions
- Sections 1.4: Counting Rules Continued
- The Binomial Experiment

Ch.1 # 56) A couple has two children.

Find the probability that both are girls given that the oldest is a girl. (Define the sample space and events.)

Find the probability that both are girls given that one of them is a girl.

Ch. 1 # 65) Show that if A and B are independent then A and B^C as well as A^C and B^C are independent.

Ch. 1 # 66) Show that ∅ is independent of A for any event A.

Section 1.4 – Counting Rules (continued)

The fundamental tools for counting are multiplication and division:

1. If there are n_i possible outcomes for each of p experiments, then there are $n_1 \times n_2 \times \ldots n_p$ total possible outcomes.

a. The # of ordered samples of size r of n distinct object with replacement is n^r

b. The number of distinct orders of n objects is $n! = n(n-1)(n-2)\ldots(2)(1)$.

2. Ordering can be removed by division.

Number of Samples of Size r	With Replacement	Without Replacement
Ordered	n^r	$\frac{n!}{(n-r)!}$ *permutation*
Unordered	$\frac{(n-1+r)!}{r!\,(n-1)!}$	$\frac{n!}{r!(n-r)!}$ *combination*

As we have seen, the Binomial Coefficient

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

is the number of distinct unordered samples of size r that can be selected from a population of size n.

It is also the number of distinct arrangements of r objects of one type and $(n-r)$ objects of another.

Example) How many distinct ways can you have 4 heads out of 10 coin flips.

This is a special case of the multinomial coefficient:

$$\binom{n}{n_1\, n_2\, \ldots\, n_r} = \frac{n!}{n_1!\, n_2! \cdots n_r!}$$

where $n = n_1 + n_2 + \ldots + n_r$.

This is the # of ways of arranging n_1 objects of type 1, n_2 objects of type 2… n_r objects of type r.

It is also the number of ways of grouping n objects into r groups of sizes $n_1, \ldots n_r$.

Example 1) Three of ten applicants are admitted to a program, and the remaining seven need to be ranked on a waiting list. How many ways can this be done?

Example 2) Ten athletes are competing for gold, silver, and bronze medals (so seven get no medal). How many distinct ways can this occur?

Example from 8/26 continued)

Components are known to have a defective rate of 0.02 (2%) and are shipped in lots of 20.

What is the probability of finding exactly 10 defectives in a lot?

In order to determine the probability of having exactly 10 out of 20 defectives we would need to have some way of easily counting the number of ways this can happen.

e.g. YYYYYYYYYYNNNNNNNNNN
YNYYYYYYYYYNNNNNNNNN
etc…

In general we get the formula:

$$P[k \text{ heads in } n \text{ flips}] = \binom{n}{k} p^k (1-p)^{n-k}$$

This applies to any situation that satisfies the conditions of being a binomial experiment.

Binomial Experiment

1. n identical trials
2. Each trial has only two possible outcomes ("Success" or "Failure")
3. Probability of "Success" is a constant p for every trial
4. Trials are independent

Why doesn't this work for opinion polls?

Hypergeometric Experiment

1. Population of size n
2. r are "successes" and $n-r$ are "failures"
3. A random sample of size k is taken without replacement