STAT 702/J702
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Today

- Examples from last time (Sections 1.2-1.3,1.5-1.6)
- Sections 1.5 Continued: Bayes' Rule
- Sections 1.4: Counting Rules

Chapter 1 – Probability *(continued)*

From last time…

- Complements: $P(A^C) = 1 - P(A)$
- Disjoint: $P(A \cup B) = P(A) + P(B)$
- Independence: $P(A \cap B) = P(A)P(B)$
- +Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- x Rule: $P(A \cap B) = P(A|B) P(B)$
  or $= P(B|A) P(A)$

Example) Last time we considered a basketball player attempting two free throws, with a 60% chance of making each one, and they were independent.

We found:

P(make, make)=36%

P(make, miss)=P(miss,make)=24%

P(miss,miss)=16%

What if the they were dependent?

Say your chance of making the first was 60%, but that your chance of making the second is 80% if you made the 1st and only 30% if you missed.

Example 2) Components are known to have a defective rate of 0.02 (2%) and are shipped in lots of 20.

a) What is the probability that the first component taken from a lot is not defective?

b) What is the probability that neither of the next two components will be defective?

c) What is the probability that the entire lot of 20 contains no defectives?

Now consider finding the probability of having exactly one defective out of 20.

d) What is the probability that the 1st component is defective and the 19 after that all work properly?

e) What is the probability that the 1st works, the 2nd is defective, and 3-18 work?

f) How many different ways can you have one defective out of 20?

g) What is the probability of exactly 1 in 20 being defective.

In order to determine the probability of having exactly 10 out of 20 defectives we would need to have some way of easily counting the number of ways this can happen.

e.g. YYYYYYYYYNNNNNNNNNN
YNYYYYYYYYNNNNNNNNN
YYNYYYYYYYNNNNNNNNN
etc…

Example 3) *Let's Make a Deal*

There are three doors… one has a car, two have livestock (you can't keep it!)

You pick a door… and I show you that another was a loser.

You can now keep your door or switch to the one I haven't shown yet.

Law of Total Probability:

Let $B_1, B_2, \ldots, B_n$ be disjoint and exhaustive so that $\cup_{i=1 \text{ to } n} B_i = \Omega$, $B_i \cap B_j = \phi$ for $i \neq j$.

Then for any A,

$P(A) = P(A|B_1)\,P(B_1) + P(A|B_2)\,P(B_2) + \cdots + P(A|B_n)\,P(B_n)$.

Example) Machines I, II, and III produce the same product.

The rates of defectives from each are 2%, 1%, and 3% respectively.

The percent of the total product made by each are 35%, 25%, and 40% respectively.

What percent of the product are defective?

Bayes' Rule: Let $B_1, \ldots, B_n$ be disjoint and exhaustive ($\cup B_i = \Omega$). Let A be any event. For any $j=1, \ldots, n$

$$P(B_j|A) = \frac{P(A|B_j)\,P(B_j)}{P(A|B_1)\,P(B_1) + \ldots + P(A|B_n)\,P(B_n)}$$

Example cont…

What is the probability that a randomly chosen defective was produced by machine 3?

Section 1.4 – Counting Rules

The fundamental tools for counting are multiplication and division:

1. If there are n_i possible outcomes for each of p experiments, then there are $n_1 x n_2 x \ldots n_p$ total possible outcomes.
   a. The # of ordered samples of size r of n distinct object with replacement is n^r
   b. The number of distinct orders of n objects is $n! = n(n-1)(n-2)\ldots(2)(1)$.
2. Ordering can be removed by division.

Number of Samples of Size r	With Replacement	Without Replacement
Ordered	n^r	$\frac{n!}{(n-r)!}$ *permutation*
Unordered	$\frac{(n-1+r)!}{r!(n-1)!}$	$\frac{n!}{r!(n-r)!}$ *combination*