STAT 702/J702
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Today

- Sections 1.2 –1.3:
  Sample Spaces & Probability
- Sections 1.5-1.6:
  Conditional Probability & Independence

Chapter 1 – Probability *(continued)*

One way of defining the probability of an event is:

The probability of an event is the proportion of times (relative frequency) that the event is expected to occur when an experiment is repeated a large number of times under identical conditions.

This is the “frequentist” idea that we will use to connect our theoretical probability constructions to reality.

Experiment – A procedure or process that produces results that cannot be predicted with certainty.

A fair coin is flipped twice.

Sample Space – The set Ω of all the possible outcomes (sample points) ω of the experiment.

{(HH), (HT), (TH), (TT)}

Sample spaces can be either discrete or continuous.

An event (usually denoted by capital letters at the beginning of the alphabet) is a set of sample points.

A=Exactly one Head = {(HT),(TH)}

B=First flip was a Head = {(HH),(HT)}

We will discuss several particular kinds of events later.

A <u>probability measure</u> on Ω is a function, P, from (events) subsets of Ω to the real numbers (0 to 1) satisfying the following three axioms:

1. $P(\Omega) = 1$
2. $P(A) \geq 0$ for any event A ($\subset\Omega$)
3. If A_1 and A_2 disjoint, then

$$P(A_1 \cup A_2) = P(A_1) + P(A_2).$$

If $A_1,\ldots,A_n,\ldots$ are <u>mutually disjoint</u>, then

$$P(A_1 \cup A_2 \cup \cdots \cup A_n \cup \cdots) = \Sigma_{i=1 \text{ to } \infty} P(A_i).$$

The coin flipping example is discrete, so we can define the probability measure by giving a probability to each of the sample points so that the sum is 1.

P(HH)=P(HT)=P(TH)=P(TT)=0.25

We would get the probability of the events *A* and *B* simply by adding the probabilities assigned to their sample points.

A=Exactly one Head = {(HT),(TH)}
P(*A*) = ¼ + ¼ = ½

B=First flip was a Head = {(HH),(HT)}
P(*B*) = ¼ + ¼ = ½

Fundamental Types of Events

- Complement- "not" A^C
- Null Event- ϕ
- Intersection- "and" $\cap$
  $A \cap B = \phi$ → "disjoint"
- Union- "or" – inclusive $\cup$

Properties of Probability (obtained from the three axioms)

A. $P(A^C) = 1 - P(A)$.
B. $P(\phi) = 0$
C. If $A \subset B$, then $P(A) \leq P(B)$.
   Proof: Since $B = A \cup (B \cap A^C)$,
   $P(B) = P(A) + P(B \cap A^C) \geq P(A)$.
D. Addition Law:
   $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Conditonal Probability

For two events A and B with P(B)>0

Define the probability of A given B as

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

A=Exactly one Head = {(HT),(TH)}
B=First flip was a Head = {(HH),(HT)}

$P(A|B) = P(A\cap B) / P(B)$

$= P(HT)/P(B)$

$= \frac{1}{4} / \frac{1}{2} = \frac{1}{2}$

Multiplication Rule

$P(A\cap B) = P(A|B)\, P(B)$
or $= P(B|A)\, P(A)$

Independence

Definition: Two events A and B are independent if

$$P(A|B)=P(A) \text{ and } P(B|A)=P(B)$$

Note that if P(B)>0 then P(A|B)=P(A) implies that P(B|A)=P(B) by the multiplication rule:

$P(A|B)=P(A)$

$\Rightarrow P(A \cap B) = P(A|B)P(B)$

$= P(A)P(B)$

$= P(B|A)P(A)$

$\Rightarrow P(B|A)=P(B)$

Definition: $A_1, \ldots, A_n$ are mutually independent events if and only if for every subcollection $A_{i1}, \ldots, A_{ik}$ of size k=2, …, n.

$$P(A_{i1} \cap A_{i2} \cap \cdots \cap A_{ik}) = P(A_{i1})\,P(A_{i2}) \cdots P(A_{ik})$$