

STAT 702/J702 – Fall 2006 - Take Home Exam 1

Due by Noon, Tuesday October 3rd

Answer 10 of the 11 following questions (I will grade your best 10). Show all of your work for credit. There are no “trick” questions, but some are decidedly easier than others.

You may not consult with anyone else on these problems; please contact me if you have any questions.

- 1) Consider the random variable X defined by the p.m.f.

x	-2	-1	0	2	5
$p(x)$	0.1	0.2	0.3	0.3	0.1

Graph the c.d.f. for X and find its mean and standard deviation.

- 2) A group of thirty students have applied for a university fellowship. Five will receive the fellowship, five must be ordered as runners-up, and the remaining twenty do not receive it. How many possible arrangements of students are possible in this situation?
- 3) Machines 1, 2, and 3 each produce 10,000 parts per day with a defective rate of 0.8%. Machines 4 and 5 each produce 15,000 parts per day with a defective rate of 1.1%. What is the probability that a randomly selected defective was produced by machine 1?
- 4) Consider events A and B with $P(A)=0.4$ and $P(B)=0.3$. Can A and B be both disjoint and independent? Justify your answer mathematically using the definitions.
- 5) A brother and sister are shooting baskets to see who gets to ride in the front seat of the car on the upcoming road trip. The brother has probability p_1 of making each shot he attempts and the sister has probability p_2 of making each shot she attempts. The first one to make it when the other misses is the winner. You may assume the shots are independent of each other. Let X =# rounds the game takes to finish. Name the distribution of X , give the value of its parameter(s), and the expected number of rounds it will take for the game to finish.
- 6) Consider taking a sample of size n from a population of size N with percentage of “successes”= p . One of the issues in deciding whether to use the binomial to approximate the hypergeometric is how different the estimated standard deviation will be. What is the largest (in terms of N and/or p) that the sample size n be so that $1 \geq \sigma_{\text{hyper}}/\sigma_{\text{binomial}} \geq 0.99$.
- 7) A batch of 80 parts will be accepted if a random sample of 10 of them contains no defectives. What is the probability that a batch containing 6 defectives would be accepted?
- 8) An assembly line produces parts with a defective rate of 0.85%. Choose an appropriate distribution to assume and estimate the number of defectives that will be found in the next 25 that are sampled.
- 9) Flaws occur in rolls of window screening at a rate of approximately 4 per 100 feet. Choose an appropriate distribution to assume and estimate the probability that no flaws will be encountered in the next 10 feet.
- 10) Consider a binomial random variable X with sample size n . Find the values of the probability p that minimize and maximize $\text{Var}(X)$ and the corresponding values of the variances. Make sure to justify your answer, including using calculus to find any potential local minima and maxima.
- 11) Consider a negative binomial random variable X with parameters p and r . Use the ratio of consecutive terms (and simplify the resulting expression) to give the formula for finding the value of k that maximizes $P[X=k]$.