STAT 702/J702 – Fall 2004 - Take Home Final Exam

Due by Noon, Wednesday, December 8th^th

Answer 10 of the 11 following questions (I will grade your best 10). Show all of your work for credit.
There are no "trick" questions and all of them relate to work we did in class.
You may not consult with anyone else on these problems; please contact me if you have any questions.

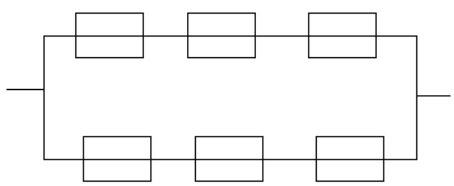
1. Assume SAT verbal scores have mean 505 and standard deviation 111, and SAT math scores have mean 511 and standard deviation 114, and the correlation between the scores is 0.479. Find the mean and standard deviation of the total scores (verbal + math).

2. Find the third and fourth moments for a standard normal random variable (you may use the formulas for the m.g.f. or p.d.f. of a normal, but must show all other work).

3. Some distributions do not have a moment generating function $M(t)=E(e^{tX})$, but do have a characteristic function $\phi(t)=E(e^{itX})$ {where i is the square root of –1}. The characteristic function has many of the same properties as a moment generating function listed in Chapter 4, including properties A, D, and the *b*X part of C. (It's versions of properties B and the remainder of C are slightly different because of the *i* however).
   The characteristic function is $\phi(t) = e^{-\sigma^2 t^2/2}$ for a normal with mean 0 and is $\phi(t) = e^{-|t|}$ for the Cauchy distribution centered at 0.

   Consider a random sample $X_1, X_2, \ldots. X_n$ from a Cauchy distribution. Use the facts above to find the distribution of $\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$.

4. Each component in the system shown below has probability *p* of failing. Find the probability that the system fails.



5. A system has 8 components whose failure time follow an exponential distribution with $\lambda=1$. The system is designed so that it will still function when one of the components fail. That is, it fails when at least 2 of the components fail. Show that the pdf of the time until the system fails is $f(t)=56[\exp(-7t)-\exp(-8t)]$

6. Consider sampling from a population of size 10,000 that could be broken into strata of sizes 5000, 3000, 1000, and 1000 respectively. Assume 10% of the population has the socio-economic trait we are interested in (the percentages in the sub-strata are 1%, 5%, 10%, and 70%). A sample of size 400 is to be taken by one of three sampling schemes: a simple random sample, a stratified random sample with 100 being sampled from each strata, and a stratified random sample with sizes 200, 120, 40, and 40 respectively. Find the variance of the estimated percentage for each of these sampling schemes.

7. Say we want to make a simple model for the total number of yards St. Louis Rams wide receiver Torry Holt will gain in a game. Assume the number of receptions follows a Poisson distribution with $\lambda=5$, and the number of yards on each reception follows a gamma distribution with $\alpha=1.2$ and $\lambda=0.08$. Find the mean and variance for the total yards per game.

8. If X_n is a sequence of χ^2 random variables each with *n* degrees of freedom, briefly explain what happens to X_n/n as $n\to\infty$.

9. Assume that a standardized test has a mean score of 511 and the standard deviation is 114, but nothing else is reported about the distribution. What is the largest percentage of students who could have scored 800 (or higher)?

10. Assume that a standardized test has a mean score of 511 and a standard deviation of 114. A random sample of 500 examinees is taken. Estimate the probability that the average test score of these 500 students will be greater than 525.

11. Consider a random sample X_1, X_2, …. X_n from a normal distribution. Use our knowledge of the t, χ^2 , and F distributions to find the distribution of $\left(\frac{\bar{x}-\mu}{s/\sqrt{n}}\right)^2$.