STAT 702/J702 – Fall 2004 - Take Home Exam 2

Due by Noon, Thursday, November 4th

Answer 10 of the 11 following questions (I will grade your best 10). Show all of your work for credit.
There are no "trick" questions, but some are decidedly easier than others.
You may not consult with anyone else on these problems; please contact me if you have any questions.

1) Let X and Y be independent random variables where X is exponential with $\lambda=2$ and Y is normal with $\mu=4$ and $\sigma=6$. Find E(X+Y) and Var(X+Y).

2) Find k such that $f(x)= kx(1-x)$ on $0<x<1$ (and is 0 elsewhere) is a p.d.f.

3) Let the random variable X have c.d.f. $F(x)= \frac{1}{2}(1 + x^3)$ on $-1\le x\le 1$ (and 0 otherwise). Find E(X) and Var(X).

4) Leaks due to manufacturing defects occur in a brand of hose at a rate of approximately 1 per 500 feet. Name an appropriate distribution and estimate the probability that the first defect will be found in the first 100 feet.

5) Evaluate $\int_0^\infty x^2 e^{-\pi x} dx$.

6) Let X have a uniform distribution on $(-\pi/2, \pi/2)$. Find the c.d.f. and p.d.f. of Y=tanX.

7) Let $f_{XY}(x,y) = \frac{1}{4} + \frac{xy}{16}$ on $-1<x<1,-1<y<1$ (and be 0 otherwise). Find the conditional distributions of X|Y and Y|X. Also, are X and Y independent?

8) Let X and Y be independent chi-square random variables with 1 degree of freedom. (The p.d.f. is on page 59.) Derive the p.d.f. of Z=X/Y.

9) In class we showed how to get general formula's for the p.d.f.'s of Z=X/Y and Z=X+Y. Show that if X and Y have joint p.d.f. $f_{XY}(x,y)$ and Z=XY that $f_Z(z) = \int_{-\infty}^{\infty} f_{XY}\left(x, \frac{z}{x}\right)\frac{1}{|x|} dx$.

10) Let X and Y have joint p.d.f. $f_{XY}(x,y) = 1 + (1-2x)(1-2y)$ on $0<x<1,0<y<1$ (and be 0 elsewhere). Find the joint p.d.f. of U=X+Y and V=X+2Y.

11) Let $X_1,\ldots X_{11}$ be independent exponential random variables with parameter $\lambda=1$. Find the p.d.f. for the median $X_{(6)}$.