STAT 530/J530
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Today

- Homework 1
- Multivariate Data
  * Data Matrix
  * Summary Parameters and Statistics
  * A Hint of Matrices
  * What if Some Data is Missing?

Homework 1

Using R, make two variables containing the ratings of "Low Energy Use", one for males and one for females.

```
mdata<-

fdata<-
```

Homework 1 Continued

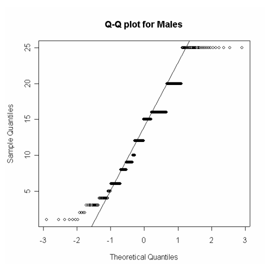
Conduct a two sample t-test to see whether there is a difference between the genders…

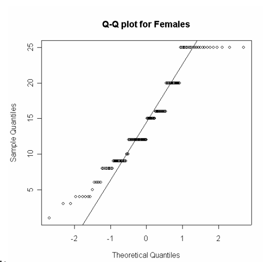
```
        Welch Two Sample t-test

data:  mdata and fdata
t = -1.7918,df = 308.021,p-value = 0.07414
alternative hypothesis: true difference in
  means is not equal to 0
```


Homework 1 Continued

…and check the assumptions using a q-q plot. Summarize your results.





Data!

Multivariate Data is typically presented in a matrix, with each of the *n* rows being a different observation, and each of the *q* columns being a different variable.

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1j} & \cdots & x_{1q} \\ x_{21} & x_{22} & & & & x_{2q} \\ \vdots & & \ddots & & & \vdots \\ x_{i1} & & & x_{ij} & & x_{iq} \\ \vdots & & & & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nj} & \cdots & x_{nq} \end{bmatrix}$$

Types of Variables

- Nominal
- Ordinal
- Interval
- Ratio

Summarizing Data

Like in univariate statistics the Mean and Variance are commonly used as summary measures.

$$\mu = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_q \end{bmatrix} \quad \sigma^2 = \begin{bmatrix} \sigma_1^2 \\ \vdots \\ \sigma_q^2 \end{bmatrix}$$

Covariances

It is also necessary to summarize the relationship between the variables, and this can be done with the covariance.

$$Cov(x_j, x_k) = E[(x_j - \mu_j)(x_k - \mu_k)]$$

$$\hat{Cov}(x_j, x_k) = \frac{1}{n-1}\sum_{i=1}^{n}(x_{ij} - \overline{x}_j)(x_{il} - \overline{x}_l)$$

Covariance Matrix

The covariance is thus a matrix:

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1q} \\ \sigma_{21} & \sigma_{11} & & \vdots \\ \vdots & & \ddots & \vdots \\ \sigma_{q1} & \cdots & \cdots & \sigma_{qq} \end{bmatrix}$$

Sample Covariance Matrix

And is estimated by the sample covariance matrix:

$$\mathrm{S} = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1q} \\ s_{21} & s_{11} & & \vdots \\ \vdots & & \ddots & \vdots \\ s_{q1} & \cdots & \cdots & s_{qq} \end{bmatrix} = \frac{\sum_{i=1}^{n} (\mathrm{x}_i - \overline{\mathrm{x}})(\mathrm{x}_i - \overline{\mathrm{x}})^T}{n-1}$$

Where $\mathrm{x}_i = \begin{bmatrix} x_{i1} \\ \vdots \\ x_{in} \end{bmatrix}$ is the i^{th} observation.

Correlation Matrix

The Covariance Matrix is commonly rescaled to be the correlation matrix:

$$\mathrm{P} = \begin{bmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1q} \\ \rho_{21} & \rho_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ \rho_{q1} & \cdots & \cdots & \rho_{qq} \end{bmatrix}$$

Where $\rho_{ij} = \dfrac{\sigma_{ij}}{\sqrt{\sigma_{ii}\sigma_{jj}}}$

Regression in Terms of Matrices

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

Sample Correlation Matrix

The Sample Correlation Matrix can be written as:

$$\mathrm{R} = \mathrm{D}^{-1/2}\mathrm{S}\mathrm{D}^{-1/2}$$

Where $D^{-1/2}$ is the matrix with $1/s_i$ on the diagonals.

What if Data is Missing?

- Missing Completely at Random?
- Imputation
- Multiple Imputation