STAT 516 - Homework 5 Solutions

1a) An ANOVA is performed to see how effective different methods of studying are, and how much extra time spent helps. Students studied by either primarily going over old exams, reading notes, or reading the text; and they studied for either two, five, or ten hours. The forty-five students were assigned at random so that there were five in each of the combinations of studying strategies and time.

Factorial - each of the nine combinations of time and method occur
With Replication - There are five observations at each level
Balanced - There are the same number (five) of observations at each level
Fixed effect probably applies for the ways of studying, but the two, five, or ten hours may or may not be random e effects (why those three time periods in particular?)

b) An ANOVA is performed to see how effective different methods of studying are, and how much extra time spent helps. Students studied by either primarily going over old exams, reading notes, or reading the text; and they studied for either two, five, or ten hours. The sixteen students were assigned at random so that four spent two hours using the text, four spent two hours using the notes, four spent two hours using the old exams, two spent five hours using text, and two spent ten hours using the text.

NOT Factorial - only five of the nine possible combinations occur
With Replication - there are two or four observations at each level that occurs
NOT Balanced - There are not the same number (it is either two or four) of observations at each level
Fixed effect probably applies for the ways of studying, but the two, five, or ten hours may or may not be random effects (why those three time periods in particular?)

c) An ANOVA is performed to see how effective different methods of studying are, and how much extra time spent helps. Students were assigned to study by either primarily going over old exams, reading notes, or reading the text; and they studied for either two, five, or ten hours. The nine students were assigned at random so that there was one in each of the combinations of studying strategies and time.

Factorial - each of the nine combinations of time and method occur
WithOUT Replication - there is only one observation at each level
Balanced - There are the same number (one) of observations at each level
Fixed effect probably applies for the ways of studying, but the two, five, or ten hours may or may not be random effects (why those three time periods in particular?)

2)

Promotional Discount	Advertising		
	None	Moderate	Heavy
	1.09	2.12	3.02
None	1.58	1.86	2.59
	2.35	3.29	4.92
	1.11	6.44	6.92
Moderate	2.69	4.25	8.52
	2.07	4.37	9.72
	3.17	10.23	21.22
Heavy	5.66	12.91	18.29
	4.59	18.84	26.77

a) It is desired to see which (if any) of the discount levels are significantly more effective than the others on average.
Holm test on all pairs of discount levels

b) It is desired to see if changing the discount level has the same effect on sales regardless of the advertising campaign expenditures.
The Type III test for interaction

c) It is desired to see if changing the level of advertising has the same effect on sales regardless of the amount of discount being offered.
The Type III test for interaction

d) It is desired to see if discounts and advertising have any affect at all on sales, or if the apparent effect is just a fluke.
The main p-value from the ANOVA table

3) The data on the web is extracted from a study that appeared in *European Bulletin of Cognitive Psychology*. Eight adopted children were selected from each of four groups. The four groups were based on two factors: adoptive parents had either very high or very low social economic status, and the biological parents had either very high or very low social economic status.

```
DATA Adopt;
INPUT IQ ADOPTIVE $              103.00     High     Low          116.00     Low      High
BIOLOGIC $;                      99.00      High     Low          113.00     Low      High
CARDS;                           125.00     High     Low          119.00     Low      High
136.00     High     High         111.00     High     Low          92.00      Low      Low
99.00      High     High         93.00      High     Low          91.00      Low      Low
121.00     High     High         101.00     High     Low          98.00      Low      Low
133.00     High     High         94.00      High     Low          83.00      Low      Low
125.00     High     High         98.00      Low      High         99.00      Low      Low
131.00     High     High         99.00      Low      High         68.00      Low      Low
103.00     High     High         91.00      Low      High         76.00      Low      Low
115.00     High     High         124.00     Low      High         115.00     Low      Low
94.00      High     Low          100.00     Low      High         ;
```

a) Write down the model equation for this two-way ANOVA being careful to identify the parameters.

$y_{ijk}=\mu_{baseline}+\alpha_i+\gamma_j+(\alpha\gamma)_{ij}+\varepsilon_{ijk}$ **for *i*=high or low, *j*= high or low, and *k*=1,...*n*, where the**

y_{ijk} are the observed IQs of the adopted children

$\mu_{baseline}$ is the baseline

α_{low}, α_{high} are the effects on the children's IQ due to the adoptive parents SES

γ_{low}, γ_{high} are the effects on the children's IQ due to the biological parents SES

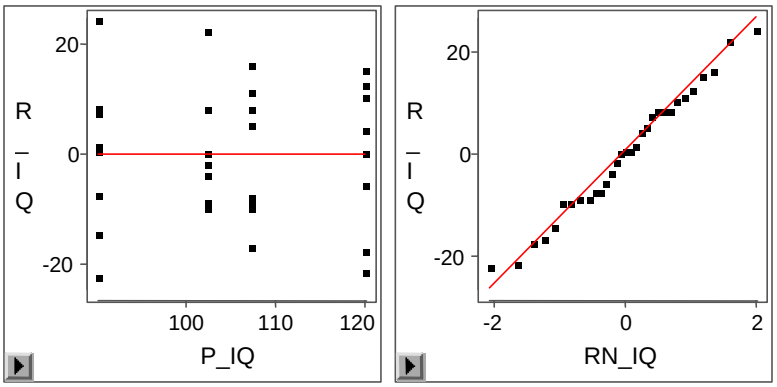
$(\alpha\gamma)_{low\text{-}low}$, $(\alpha\gamma)_{low\text{-}high}$, $(\alpha\gamma)_{high\text{-}low}$, $(\alpha\gamma)_{high\text{-}high}$ are the interactions for the combinations of adoptive and biological parents SES

ε_{ijk} are the errors

b) Check the assumptions for the two-way ANOVA... including using Levene's test for the variances. Also, comment on why it is unreasonable to expect data of this sort to have the independence you would usually find in an experiment.

Using PROC INSIGHT to get the plots...

```
PROC INSIGHT;
OPEN Adopt;
FIT IQ=ADOPTIVE BIOLOGIC ADOPTIVE*BIOLOGIC;
RUN;
```



We see that the errors appear to be approximately normally distributed from the Q-Q plot. The means of the errors are zero for each of the groups, although one of the groups appears skewed. Also, there may be some problems with the variances being equal.

Using PROC GLM (as per the web-site) to test whether the variances are equal...

```
DATA Adopt2;
SET Adopt;
KEEP  Block IQ;
Block = trim(ADOPTIVE)||trim(BIOLOGIC);

PROC GLM DATA=Adopt2 ORDER=DATA;
CLASS Block;
MODEL IQ = Block;
MEANS Block / HOVTEST=BF;
RUN;
```

Brown and Forsythe's Test for Homogeneity of IQ Variance
ANOVA of Absolute Deviations from Group Medians

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Block	3	62.3438	20.7813	0.37	0.7762
Error	28	1578.4	56.3705		

With a p-value of 0.7762 we fail to reject the null hypothesis that the variances are equal.

We probably would not expect the errors to be independent because those of higher SES who are looking to adopt have many more options than those of low SES, just as those of higher SES who are giving there children up for adoption have many more options. (Not to mention the associations in modern American society between poverty and race, and ease of adoption and race.)

c) State what hypotheses are being tested by the p-value in the ANOVA table and the p-value in the Type III tests, in terms of the parameters you wrote down in part A. Also describe what each of your conclusions mean in a brief sentence or two.

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Stat	Pr > F
Model	3	3730.8438	1243.6146	7.48	0.0008
Error	28	4655.3750	166.2634		
C Total	31	8386.2188			

Type III Tests

Source	DF	Sum of Squares	Mean Square	F Stat	Pr > F
ADOPTIVE	1	1262.5313	1262.5313	7.59	0.0102
BIOLOGIC	1	2467.5313	2467.5313	14.84	0.0006
ADOPTIVE*BIOLOGIC	1	0.7812	0.7812	4.699E-03	0.9458

The ANOVA table p-value of 0.0008 means we reject the null hypothesis that $\alpha_{low} = \alpha_{high}$, $\gamma_{low}=\gamma_{high,}$ and $(\alpha\gamma)_{low\text{-}low}= (\alpha\gamma)_{low\text{-}high}= (\alpha\gamma)_{high\text{-}low}=(\alpha\gamma)_{high\text{-}high}$. That is, at there is some effect on the IQ of adopted children based on the SES of the adoptive and/or SES biological parents and/or the interaction of the two.

The first Type III p-value of 0.0102 means that we reject the null hypothesis that $\alpha_{low} = \alpha_{high}$. That is, there is an effect on the IQ of adopted children based on the SES of the adoptive parents.

The second Type III p-value of 0.0006 means that we reject the null hypothesis that $\gamma_{low}=\gamma_{high}$. That is, there is an effect on the IQ of adopted children based on the SES of the biologic parents.

The last Type III p-value of 0.9458 means that we fail to reject the null hypothesis that $(\alpha\gamma)_{low\text{-}low}= (\alpha\gamma)_{low\text{-}high}= (\alpha\gamma)_{high\text{-}low}=(\alpha\gamma)_{high\text{-}high}$. That is, there is no evidence of an interaction between the SES of the adoptive and biological parents in regards to the IQ of the adopted children.

d) Construct an appropriate display to illustrate the main effects and interactions.

```
PROC GLM DATA=adopt ORDER=DATA;
CLASS adoptive biologic;
MODEL iq=adoptive biologic adoptive*biologic;
MEANS adoptive*biologic;
RUN;
```

Level of ADOPTIVE	Level of BIOLOGIC	N	----IQ------ Mean
High	High	8	120.375000
High	Low	8	102.500000
Low	High	8	107.500000
Low	Low	8	90.250000