STAT 515 - Spring 2003 - Solutions to the Practice Homework

Pg. 186: 4.22 part a only

Using the standard formulas (page 183 and 184) gives

$\mu=\Sigma x\, p(x)= 10(0.05)+20(0.20)+30(0.30)+40(0.25)+50(0.10)+60(0.10)$

$= 0.5+4.0+9.0+10.0+5.0+6.0=34.5$

$\sigma^2=\Sigma (x-\mu)^2\, p(x)$

$=(10-34.5)^2 0.05 + (20-34.5)^2 0.20 + (30-34.5)^2 0.30 + (40-34.5)^2 0.25 + (50-34.5)^2 0.10 + (60-34.5)^2 0.10$

$=(-24.5)^2 0.05 + (-14.5)^2 0.20 + (-4.5)^2 0.30 + (5.5)^2 0.25 + (15.5)^2 0.10 + (25.5)^2 0.10$

$=174.75$

$\sigma=\sqrt{\sigma^2} = \sqrt{174.75} = 13.21930$

Pg. 201: 4.52, also a new part f and g

a) If the psychic is guessing (no ESP) what is the value of p, the probability of a correct decision on each trial? **Only 1 out of 10 boxes is correct, so 1/10.**

b) If the psychic is guessing, what is the expected number of correct decisions in seven trials? **This is a binomial experiment with p=0.1 and n=7, so $\mu=np=7(0.1)=0.7$ correct decisions**

c) IF the psychic is guessing, what is the probability of no correct decisions in seven trials? **This is a binomial experiment with p=0.1 and n=7, the problem is asking $P(X=0)=(n \text{ choose } x)p^x(1-p)^{n-x}$ $= (7 \text{ choose } 0)0.1^0(1-0.1)^7 = 1\ (1)\ (0.9)^7 = 0.4782969$ You could also answer this one by using just the multiplication rule, and you would end up with the same $(0.9)^7$.**

d) Now suppose the psychic has ESP with p=0.5. What is the probability of no correct decisions in seven trials? **Same as in c, but with a new p. $P(X=0)= (7 \text{ choose } 0)0.5^0(1-0.5)^7 = 1\ (1)\ (0.5)^7 = 0.0078125$**

e) If the psychic failed on all seven trials, is this evidence against them having ESP? Explain. **Yes. The chance of them getting none correct if they really have ESP of any worth is only 7 out of a 1,000. That isn't very likely!**

f) What is the probability that the psychic would get exactly two correct if they had no ESP? **This one would be really annoying to do out the long way, but is easy if you use the binomial formula. $P(X=2)= (7 \text{ choose } 2)0.1^2(1-0.1)^5 = 7!/(2!5!)\ (0.1)^2(0.9)^5 = 21\ (0.1)^2(0.9)^5 = 0.1240029$**

g) What is the probability that the psychic would get exactly two correct if they had ESP with p=0.5? **$P(X=2)= (7 \text{ choose } 2)0.5^2(1-0.5)^5 = 21\ (0.5)^2(0.5)^5 = 0.1640625$**

Page 234: 5.16 a,d; 5.20 d; 5.24 c

5.16a) Want $P(z>1.46)$

The table gives $P(0<z<1.46)=0.4279$

These two add up to 0.5000 (draw the picture!) so we get 0.5000-0.4279=**0.0721**

5.16d) Want $P(-1.96<z<-0.33)$

The table give $P(0<z<1.96)=0.4750$ and $P(0<z<0.33)=0.1293$

The area we want is the difference between these two (draw the picture!) so we get 0.4750-0.1293=**0.3457**

5.20d) Want z_0 so that $P(-z_0<Z<z_0)=0.8740$

The table would give us an area of the form $P(0<Z<z_0)$, this is half the area of the one we want (draw the picture!) so we need to look up the area 0.8740/2=0.4370. This gives $z_0=$**1.53.**

5.24c) $P(13<X<16)$ where $\mu=11$ and $\sigma=2$

We first need to change this from X to Z using $Z=(X-\mu)/\sigma$. What we want is:

$$P(13 < X < 16) = P\left(\frac{13-11}{2} < \frac{X-\mu}{\sigma} < \frac{16-11}{2}\right) = P(1 < Z < 2.5)$$

The table gives $P(0<Z<1)=0.3413$ and $P(0<Z<2.5)=0.4938$.

The value we want is the difference of these two (draw the picture!) so we get **0.1525**.