**Statistics 515 - Fall 2002 - Exam 2** *(slightly modified)* **Answers**

Part I: Answer seven of the following eight questions. If you complete more than seven, I will grade only the first seven. Five points each.

1) $P(X \leq 45) = P(X \leq 45.5) = P\left( \dfrac{X - np}{\sqrt{np(1-p)}} \leq \dfrac{45.5 - 51}{\sqrt{100(.51)(.49)}} \right) \approx P(Z \leq -1.10022) = .5 - .3643 = .1357$

2) Define what is meant by "$\alpha$-level". **The probability of rejecting the null hypothesis if it is true. -or- The probability of committing a Type I error if the null hypothesis is true.**

3) Define what is meant by "the p-value (or observed significance level) of a test".
**The probability of observing a test statistic at least as extreme as the one observed if the null hypothesis is true.**

4) A $(1-\alpha)100\%$ confidence interval becomes **narrower** if the sample size is increased, and **narrower** if $\alpha$ increases

5) A sample of size 20 results in $\bar{x} = 10.0$ and $s = 2.4$. Assume that the necessary assumptions are met and construct a 95% CI for $\sigma^2$. **Using the formula and df=19.** $\left( \dfrac{19(2.4)^2}{32.8523}, \dfrac{19(2.4)^2}{8.90655} \right) = (3.33, 12.3)$

6) $\hat{p}_1$ **and** $\hat{p}_2$ **for the confidence interval because we don't know** $p_1$ **or** $p_2$, $\bar{p}$ **for the test because we are assuming the proportions are the same and can pool them together.**

7) **Trust the t-test, don't trust the** $\chi^2$ **or F test**

8) At $\alpha=0.10$ we **accept** the null hypothesis that $\sigma^2_N = \sigma^2_S$. At $\alpha=0.01$ we **accept** the null hypothesis that $\sigma^2_N = \sigma^2_S$.

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Equality of Variances

| Variable | Method   | Num DF | Den DF | F Value | Pr > F |
|----------|----------|--------|--------|---------|--------|
| value    | Folded F | 9      | 11     | 2.57    | **0.1419** |

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Part II: Answer every part of the next two problems. Read each problem carefully, and show your work for full credit. Twenty points each.

1) A manufacturer of alkaline batteries wants to be reasonably certain that fewer than 5% of its batteries are defective before each manufacturing run is shipped. To check this a random sample of 300 batteries will be selected from each manufacturing run and tested.

A) State the appropriate null and alternate hypotheses for testing if the broker will be able to ship the current manufacturing run of batteries.

**Let p=the proportion of batteries in the entire run that are defective.**
**$H_0$: p=0.05 (don't ship)**
**$H_A$: p<0.05 (ship)**

B) A sample of size 300 is acquired and it 10 defective batteries are found. Test the hypothesis in A at an $\alpha=0.01$ level and state whether the manufacturer should ship this manufacturing run or not.

$$z = \frac{\hat{p} - p}{\sqrt{\dfrac{p(1 - p)}{n}}} = \frac{\dfrac{10}{300} - 0.05}{\sqrt{\dfrac{0.05(1 - 0.05)}{300}}} = \text{-1.32}$$ **We compare this to -2.326 and fail to reject H₀... do not ship!**

C)  Besides the sample being randomly chosen, what other assumption(s) are required to trust the test in part B?  If possible, check that the assumption(s) hold.  **The sample size must be large enough... np=15≥5 and n(1-p)=285≥5**

2) A psychologist wishes to compare the spatial geometry ability of male and female college students.  To do this a random sample of eighty men and eighty women are selected from students at USC and nagged until they consent to participate in the experiment.  Each student is then given a score ranging from 0 to 100 (higher is more ability).   The male subjects had an average score of 57.4, a standard deviation of 18.4  (variance=338.56), and seemed to be from a population that was slightly skewed left.  The female subjects had an average score of 44.5 a standard deviation of 8.7 (variance=75.69) and seemed to be from a population that was approximately normally distributed.

A) State the appropriate null and alternate hypothesis for determining whether there is a difference in the spatial geometry ability of male and female college students.

**$\mu_m$ = average score of all college men**
**$\mu_f$ = average score of all college women**
**$H_0$: $\mu_m = \mu_f$  or  $\mu_m - \mu_f = 0$ (no difference)**
**$H_A$: $\mu_m \neq \mu_f$  or  $\mu_m - \mu_f \neq 0$  (difference)**

B) Conduct the appropriate test of the hypothesis in part A, and report whether there is a significant difference in their spatial geometry ability at an $\alpha$=0.10 level.

**Because the sample sizes are large, we just plug in the sample variances and use the z.**

$$z = \frac{\bar{x}_m - \bar{x}_f - (\mu_m - \mu_f)}{\sqrt{\dfrac{s_m^2}{n_m} + \dfrac{s_f^2}{n_f}}} = \frac{57.4 - 44.5 - (0)}{\sqrt{\dfrac{338.56}{80} + \dfrac{75.69}{80}}} = 5.669$$

**We compare this to 1.645 and reject H₀.  There is a difference between the average spatial geometry skills of male and female college students.**

C) Construct a 95% confidence interval for this difference.  Does a positive difference mean that men are more able, or that women are more able?

$$\bar{x}_m - \bar{x}_f \pm z_{\alpha/2} \sqrt{\dfrac{s_m^2}{n_m} + \dfrac{s_f^2}{n_f}} \qquad (57.4 - 44.5) \pm 1.96 \sqrt{\dfrac{338.56}{80} + \dfrac{75.69}{80}} \qquad 12.9 \pm 4.46$$

**A positive difference means that men are more able.**