Statistics 515 - Fall 2003 - Practice Exam 3 Solutions

Part I:

1) Define what is meant by the *p-value* (or the *observed significance level*) of a test. **The probability of observing a test statistic at least as extreme as the one observed if the null hypothesis is true.**

2) When conducting a two-sample t-test that two population means are equal we use $\underline{\mathbf{s_p^2}}$ and **<u>n_1+n_2-2 degrees of freedom</u>** if the variances are equal.

3) If you knew nothing else about the student, regression to the mean would imply that they would score **<u>lower</u>** if they retook the test. A student scoring a 500 out of 800 (near the average) would score **<u>about the same</u>** if they retook it.

4) Briefly explain why can't we automatically assume that changing *x* causes *y* to change? **It has to be an experiment to show causation, they could both be related to another variable (like the storks and people related to the natural disaster).**

Part II:

1a) State the appropriate null and alternate hypothesis for determining whether the candidate differs in popularity between men and women. Be sure to identify what the symbols you are using mean in terms of the problem. **H_0: $p_m=p_f$ (equal popularity) vs. H_A: $p_m \neq p_f$ (different popularity) where p_m is the percentage of men in the population who would vote for the candidate and p_f is the percentage of women.**

b) Of those sampled, 105 of the men and 128 of the women plan on voting for the candidate. Report the p-value for the test of hypothesis in A.

$$\hat{p}_m = \frac{105}{250} = 0.420 \qquad \hat{p}_f = \frac{128}{250} = 0.512 \qquad \bar{p} = \frac{n_m \hat{p}_m + n_f \hat{p}_f}{n_m + n_f} = \frac{105+128}{250+250} = 0.466$$

$$z = \frac{(\hat{p}_m - \hat{p}_f) - (p_m - p_f)}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_m} + \frac{\bar{p}(1-\bar{p})}{n_f}}} = \frac{(0.420 - 0.512) - 0}{\sqrt{\frac{0.466(1-0.466)}{250} + \frac{0.466(1-0.466)}{250}}} = \frac{-0.092}{0.0446} = -2.06$$

Comparing this to a z table we get that the area below -2.06 is 0.5 - 0.4803= 0.0197, doubling this we get the p-value is 0.0394.

c) Besides the sample being randomly chosen, what other assumption(s) are required to trust the test in part B? <u>If possible</u>, check that the assumption(s) hold.

The sample size needs to be large enough. $n_m \hat{p}_m = 105, n_m(1-\hat{p}_m) = 145, n_f \hat{p}_f = 128, n_f(1-\hat{p}_f) = 122$

are all greater than 5.

2a) Complete the above ANOVA table by filling in the missing values.

Source	SS	DF	MS	F	p-value
Treatments	3.3054	**2**	**1.6527**	**25.35**	0.000
Error	1.9553	30	**0.0652**		
Total	**5.2608**	32			

b) How many different headache remedies were compared in this experiment? **df for treat=p-1 so p-1=2 so p=3**
How many total patients were used in this experiment? **df for total=n-1 so n-1=32 so n=33**

c) What null and alternate hypothesis are being tested by the p-value in this ANOVA table? **H_0: $\mu_1=\mu_2=\mu_3$ vs. H_A: not all equal where μ_i the mean number of hours for headache remedy *i* to take effect.**

d) Do the headache remedies provide the same average relief, or is there a difference? (How did you decide?) **Different. p-value=0.000<α so reject the null hypothesis.**

3a) Say which three assumptions those are and why they seem to be met in this case.
Errors are normal - q-q plot looks like a straight line
Errors have mean zero - in the residual vs. predicted plot, zero is in the middle at each value of P_interval
Errors have constant variance - in the residual vs. predicted plot the values do not spread out that much moving from left to right

b) Assuming the assumptions of the regression model are met, what is the p-value for testing the hypothesis that $\beta_1 = 0$? Do we accept or reject this null hypothesis at $\alpha = 0.01$? **p-value is <.0001 and we reject the null hypothesis. Duration does predict interval.**

c) If old faithful just erupted for 3 minutes, how long do you predict it will be until the next eruption?
interval = 33.8212 + 10.7412(3) = 66.0448 minutes

d) What is the estimate of the standard deviation of the errors (σ) for this regression? **root-MSE = 6.6815 minutes**

e) What percent of the variation or error in predicting the time until the next eruption is explained by the duration of the previous eruption? **r^2 = 0.737 so 73.7%**