STAT 515 - Spring 2003 - Exam 2 - *(Modified for Fall 2003 Practice)* - Solutions

Part I: 1) X is a normal random variable with $\mu=25$, $\sigma^2=25$, and $\sigma=5$. Find $P(X \le 30)$. **= P((X-μ)/σ ≤ (30-25)/5) = P(Z ≤ 1) = 0.5+0.3413=0.8413 (Draw the picture!)**

2) Z is a standard normal random variable. Find z_0 such that $P(Z \ge z_0) = 0.0778$. **Look up 0.4222 on the table to get 1.42 (Draw the Picture!)**

3) Because $E(\bar{x})=\mu$, the **statistic** $\bar{x}$ **is** an unbiased estimate of the **parameter** μ.

4) Suppose that a circuit board factory produces 6% defective products. Use the central limit theorem to approximate the probability that 20 or more defectives are observed in a sample of 200. **This is Example 5.12 on page 247**.

$$P\left(\frac{Y-np}{\sqrt{np(1-p)}} \ge \frac{19.5-200(0.06)}{\sqrt{200(0.06)(1-0.06)}}\right) = P\left(Z \ge 2.23\right) = 0.5 - 0.4871 = 0.129$$

5) Define what is meant by "Type II error". **Failing to reject a false null hypothesis.**

6) Define what is meant by "the p-value (or observed significance level) of a test". **The probability of observing a test statistic at least as extreme as the one observed if the null hypothesis is true.**

7) A sample of size 20 gives $\bar{x}=18.45$ and $s=1.84$. Construct a 95% confidence interval for μ.

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} \quad \Rightarrow \quad \bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} \quad \Rightarrow \quad 18.45 \pm 2.093\frac{1.84}{\sqrt{20}} \quad \Rightarrow \quad 18.45 \pm 0.86 \quad \Rightarrow \quad (17.59, 19.31)$$

8) If you increase the sample size, you will likely **decrease** the width of a confidence interval. If you increase the confidence coefficient $(1-\alpha)$ you will **increase** the width.

Part II: 1A) State the appropriate null and alternate hypotheses for testing if the manufacturer will be allowed to ship the current batch of anesthesia. **H_0: σ=0.05 (don't ship) H_A: σ<0.05 (ship) where σ is the standard deviation.**

B) A sample of size 16 is obtained. The sample has standard deviation 0.04. Test the hypothesis in A at an $\alpha=0.10$ level and state whether the manufacturer should ship this run or not.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{15(0.04)^2}{(0.05)^2} = 9.6$$ **but since the rejection region is 8.54675 or less, we fail to reject H_0. Don't ship.**

C) Besides the sample being randomly chosen, what other assumption(s) are required to trust the test in part B? If possible, check that the assumption(s) hold. If it is not possible from what is given, say what you would need to have in order to check the assumption(s). **We need a q-q plot to check if the population appears to be normally distributed.**

2A) State the appropriate null and alternate hypotheses for testing whether the HMO will remove the doctor from their list of preferred providers. **H_0: p=0.75 (don't remove) H_A: p<0.75 (remove) where p is the percentage of all patients who did not get either "excellent" or "very-good" service.**

B) The 100 selected files for one physician revealed 69 patients reporting "excellent" or "very-good" service. Report the p-value for this data for testing the hypotheses you gave in part A. In addition, state your conclusion at the $\alpha=0.05$ level.

$$z = \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.69-0.75}{\sqrt{\frac{0.75(1-0.75)}{100}}} = -1.39$$

This is problem 8.62 from the practice homework.
The area less than this is 0.5-0.4177=0.0823.
We fail to reject the null hypothesis and they are not removed.

C) Besides the sample being randomly chosen, what other assumption(s) are required to trust the test in part B? If possible, check that the assumption(s) hold. If it is not possible from what is given, say what you would need to have in order to check the assumption(s). **np=75≥5 and n(1-p)=25≥5 so the assumptions are met.**