Practice Homework #2 Solutions

Pg. 352: 8.57 H_0: μ=0.100 H_A: μ>0.100 α=0.01

$$t_{df=n-1=7} = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{0.210-0.100}{\sqrt{0.011}/\sqrt{8}} = 2.97$$

The rejection region is >2.998, so we fail to reject the null hypothesis. We cannot conclusively say it exceeds the limits.

Pg. 357: 8.62 H_0: p=0.75 H_A: p<0.75 α=0.05

a) Since 0.69 is less than 0.75 it does seem to contradict the null hypothesis.

b) $$z = \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.69-0.75}{\sqrt{\frac{0.75(1-0.75)}{100}}} = -1.39$$

The rejection region is <-1.645, so we fail to reject the null hypothesis. We find that while it looks like p<0.75, we do not have enough evidence to decide this conclusively.

c) We need to find the area less than z=-1.39. The table gives that the area between 0 and 1.39 is 0.4177, so the p-value is 0.823 (draw the picture!). There is an 8.23% chance we would observe this much evidence against the null hypothesis, even if it were true. This does not meet the 5% standard we set.

Pg. 370: 8.88

a) We must assume that the population is normally distributed (check it using a q-q plot).

b) H_0: σ^2=1 H_A: σ^2>1 α=0.05

$$\chi^2_{df=n-1=6} = \frac{(n-1)s^2}{\sigma^2} = \frac{(7-1)4.84}{1} = 29.04$$

The rejection region is >12.5916, so we reject the null hypothesis and conclude the variance is greater than one.

c) H_0: σ^2=1 H_A: $\sigma^2\neq$1 α=0.05

$$\chi^2_{df=n-1=6} = \frac{(n-1)s^2}{\sigma^2} = \frac{(7-1)4.84}{1} = 29.04$$

The rejection region is <1.237347 or >14.4494, so we reject the null hypothesis and conclude the variance is not equal to one.

Pg. 392: 9.10

a) The output shows a p-value of 0.1114 so we would not reject the null hypothesis even if the a-level was 0.10. (Note SAS always gives the p-value for ≠, so we would have needed to draw the picture and figure out the p-value manually if the alternate hypothesis was < or >).

To check the numbers we first see that since we were testing H_0:(μ_1-μ_2)=0 vs. H_A:(μ_1-μ_2)≠0 we need the test statistic that compares two means:

$$\frac{\bar{x}_1-\bar{x}_2-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$$

But, of course we don't know σ^2_1 or σ^2_2. Since the sample sizes are small (17 and 12) we need to assume the variances are equal and use the pooled variance:

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{(17-1)3.4^2 + (12-1)4.8^2}{17+12-2} = 16.237$$

And so,

$$t_{df=n_1+n_2-2=27} = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1-\mu_2)}{\sqrt{\frac{s_p^2}{n_1}+\frac{s_p^2}{n_2}}} = \frac{5.4-7.9-0}{\sqrt{\frac{16.237}{17}+\frac{16.237}{12}}} = \frac{-2.5}{1.519} = -1.646$$

b) We get the formula for the confidence interval in the usual way:

$$t = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1-\mu_2)}{\sqrt{\frac{s_p^2}{n_1}+\frac{s_p^2}{n_2}}} \quad \Rightarrow \quad \bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2}\sqrt{\frac{s_p^2}{n_1}+\frac{s_p^2}{n_2}} \quad \Rightarrow \quad -2.5 \pm 2.052(1.519) \text{ or } (-5.617, 0.617)$$

Note that this is different than the value shown in the computer output!!

<u>Pg. 415: 9.51</u>

a) 14/98=0.1429

b) 5/102=0.0490

c) H_0: p_{sj}=pp or $p_{sj}-p_p=0$ H_A: $p_{sj}>p_p$ or $p_{sj}-p_p>0$

The formula for two percentages is:

$$z = \frac{\hat{p}_1 - \hat{p}_2 - (p_1 - p_2)}{\sqrt{\frac{p_1(1-p_1)}{n_1}+\frac{p_2(1-p_2)}{n_2}}}$$

but we don't have the value for p_1 and p_2 to put in the denominator, so we need to use

$$\bar{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1+n_2} = \frac{98(.1429)+102(.0490)}{98+102} = 0.095$$

which gives

$$z = \frac{\hat{p}_1 - \hat{p}_2 - (p_1 - p_2)}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_1}+\frac{\bar{p}(1-\bar{p})}{n_2}}} = \frac{0.1429-0.049-(0)}{\sqrt{\frac{0.095(1-0.095)}{98}+\frac{0.095(1-0.095)}{102}}} = \frac{0.0939}{0.041475} = 2.26$$

At α=0.01, we compare this to 2.326 and fail to reject H_0 (we conclude St. John's wort does not work).

d) At α=0.10, we compare this to 1.282 and reject H_0 (we conclude St. John's wort does work).

e) α is important because it changes our conclusion!