Answers to the hypothesis testing practice problems...

1a) H_0: $\mu=500$ (does not comply) vs. H_A: $\mu<500$ (does comply). We are looking for evidence that it does comply, and so that will be are alternate hypothesis.

b) A type I error would be saying it did comply when it really did not. A type II error would be saying it did not comply when it really did.

c) $$t=\frac{\bar{x}-\mu}{s/\sqrt{n}}=\frac{402.1-500}{112.2/\sqrt{25}}=-4.36$$

We reject H_0 as the test statistic is less than -1.711 (the value for the lower 5% for a t-distribution with 24 degrees of freedom. Therefore we conclude the plant does comply.

d) $$\bar{x}\pm t_{\alpha/2}\,s/\sqrt{n}=402.1\pm 1.711\,112.2/\sqrt{25}=402.1\pm 38.4=(363.7,440.5)$$

e) The sample needs to be random and the population needs to be normally distributed. The t-test and confidence interval for a mean are fairly robust against violations of these assumptions however.

2a) If we let p=% of times a fair die will show a 6 then: H_0: $p=1/6$ (fair) vs. H_A: $p\neq 1/6$ (not fair)

b) Yes, $50(1/6)\approx 8.33\geq 5$ and $50(1-1/6)\approx 41.67\geq 5$

c) $$z=\frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}}=\frac{\frac{10}{50}-\frac{1}{6}}{\sqrt{\frac{\frac{1}{6}\left(1-\frac{1}{6}\right)}{50}}}=+0.632$$

The area below +0.63 is (0.5-0.2357)=0.2643 and twice that is the p-value=0.5286. Why twice? See page 336.

d) As the p-value is greater than the α-level of 0.01 we fail to reject the null hypothesis. We do not have significant evidence that the die is unfair.

3a) H_0: $\sigma^2=0.54$ (theory true) vs. H_A: $\sigma^2>0.54$ (theory false) We want evidence that the theory is not true, so that must be the alternate hypothesis. A type I error would be saying the theory is false when it is actually true. A type II error would be accepting the theory as true when it is really false.

b) $$\chi^2=\frac{(n-1)s^2}{\sigma^2}=\frac{(41-1)0.5513}{0.54}=40.83$$

We compare this value to the upper 5% of the chi-square distribution with df=n-1=41-1=40. This value is 55.7585. Because the test statistic is not in the rejection region we fail to reject the null hypothesis. At $\alpha=0.05$ we do not have significant evidence against the theory.

c) We can't get the exact p-value here because the chi-squared table doesn't give us enough values. Looking at the chi-squared table we see that 40.83 is between 51.8050 (goes with a p-value of 0.10) and 29.0505 (goes with a p-value of 0.90). So all we can say about the p-value is that it is between 0.10 and 0.90. We thus know we would definitely fail to reject the null hypothesis for any α-level 0.10 or less. Using the table we can't tell for any of the other values.

d) The sample needs to be random and the population needs to be normally distributed. The chi-squared test and confidence interval for a variance are not very robust against violations of these assumptions however.