Answers to the hypothesis testing worksheet of 10/18/01

1a) H_0: μ=500 (does not comply) vs. H_A: μ<500 (does comply). We are looking for evidence that it does comply, and so that will be are alternate hypothesis.

b) A type I error would be saying it did comply when it really did not. A type II error would be saying it did not comply when it really did.

c) $$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{402.1-500}{112.2/\sqrt{25}} = -4.36$$

We reject H_0 as the test statistic is less than -1.711 (the value for the lower 5% for a t-distribution with 24 degrees of freedom. Therefore we conclude the plant does comply.

d) $$\bar{x} \pm t_{\alpha/2}\, s/\sqrt{n} = 402.1 \pm 1.711\, 112.2/\sqrt{25} = 402.1 \pm 38.4 = (363.7, 440.5)$$

e) The sample needs to be random and the population needs to be normally distributed. The t-test and confidence interval for a mean are fairly robust against violations of these assumptions however.

2a) If we let p=% of times a fair die will show a 6 then: H_0: p=1/6 (fair) vs. H_A: p≠1/6 (not fair)

b) Yes, $50(1/6) \approx 8.33 \geq 5$ and $50(1-1/6) \approx 41.67 \geq 5$

c) $$z = \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{\frac{10}{50}-\frac{1}{6}}{\sqrt{\frac{\frac{1}{6}\left(1-\frac{1}{6}\right)}{50}}} = -0.632$$

The area below -0.63 is (0.5-0.2357)=0.2643 and twice that is the p-value=0.5286. Why twice? See page 336.

d) As the p-value is greater than the α-level of 0.01 we fail to reject the null hypothesis. We do not have significant evidence that the die is unfair.

3a) H_0: σ^2=0.54 (theory true) vs. H_A: σ^2>0.54 (theory false) We want evidence that the theory is not true, so that must be the alternate hypothesis. A type I error would be saying the theory is false when it is actually true. A type II error would be accepting the theory as true when it is really false.

b) $$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(41-1)0.5513}{0.54} = 40.83$$

We compare this value to the upper 5% of the chi-square distribution with df=n-1=41-1=40. This value is 55.7585. Because the test statistic is not in the rejection region we fail to reject the null hypothesis. At α=0.05 we do not have significant evidence against the theory.

c) We can't get the exact p-value here because the chi-squared table doesn't give us enough values. Looking at the chi-squared table we see that 40.83 is between 51.8050 (goes with a p-value of 0.10) and 29.0505 (goes with a p-value of 0.90). So all we can say about the p-value is that it is between 0.10 and 0.90. We thus know we would definitely reject the null hypothesis for any α-level 0.10 or less. Using the table we can't tell for any of the other values.

d) The sample needs to be random and the population needs to be normally distributed. The chi-squared test and confidence interval for a variance are not very robust against violations of these assumptions however.

4) This question was put on in case a few people were completing the work sheet too quickly. It is related to the material in section 8.6. It is a little tricky to look at as one big problem, but each individual part should be very straight forward.

a) If the two sided hypothesis has α=0.01 that means 0.005 is in each tail (or end) of the normal curve. That would mean the rejection region is all z values less than -2.576 and all z values greater than +2.576.

b) To find the rejection region in terms of $\hat{p}$ we simply need to solve the equation in reverse.

$$\pm 2.576 = \frac{\hat{p} - \frac{1}{6}}{\sqrt{\frac{\frac{1}{6}\left(1-\frac{1}{6}\right)}{50}}} \quad \text{so} \quad \hat{p} = \frac{1}{6} \pm 2.576\sqrt{\frac{\frac{1}{6}\left(1-\frac{1}{6}\right)}{50}} = 0.0309 \text{ to } 0.3024$$

c) Just multiply by 50. (Remember $\hat{p} = \frac{\#obs}{n}$ so $\#obs = \hat{p}n$.) So, 1.545 to 15.12. Because we can't observe a fraction of a six, we would reject if it was less than or equal to 1 or greater than or equal to 16.

d) This is just the complement of the rejection region, so we fail to reject for any observation between 2 and 15.

e) This is just a flashback to section 5.5 and the last problem on exam 1:

$$P(2 \le X \le 15) \approx P\left(\frac{1.5 - 50(0.25)}{\sqrt{50(0.25)(1-0.25)}} \le \frac{X - np}{\sqrt{np(1-p)}} \le \frac{15.5 - 50(0.25)}{\sqrt{50(0.25)(1-0.25)}}\right)$$

$$= P\left(\frac{-11}{\sqrt{9.375}} \le Z \le \frac{3}{\sqrt{9.375}}\right) = P(-3.59 \le Z \le 0.98) = 0.5 + 0.3365 = 0.8365$$

So we would have an 83.65% chance of being in the not-rejection region if p was really 0.25 instead of 1/6. So, we would have an 83.65% chance of committing a Type II error in this case!!!!!!