1) A company is ordered to reduce the mean discharge from their plant into a nearby river to less than 500 manganate parts per million. Does the company have significant evidence to show that it complied?

a) What null and alternate hypothesis should be tested?

b) What would a type I error be for this problem? A type II error?

c) A sample of size 25 revealed values between 162.8 and 635.4. The mean concentration observed was 402.1 and the observed standard deviation was 112.2. Perform the test of the hypothesis in a at an α-level of 0.05.

d) Say it was desired to construct a confidence interval instead. Construct a 90% confidence interval for the true mean discharge.

e) What assumptions do you need to make for parts b and c? Is this test robust against the violations of these assumptions?

2) A 6 should show one out of six times a fair die is rolled. To test if a die is fair it is rolled fifty times.

a) What null and alternate hypotheses should be tested to see if the die is fair?

b) Is the sample size large enough to conduct a test of these hypotheses?

c) Ten sixes are observed out of the fifty. What is the p-value for performing this test?

d) Do you accept or reject the null hypothesis at an α-level of 0.01?

3) To improve the signal-to-noise ratio (SNR) in the electrical activity of the brain, neurologists repeatedly stimulate subjects and average the responses - a procedure that assumes that single responses are homogeneous. A study was conducted to test this homogeneous signal theory (*IEEE Engineering in Medicine and Biology Magazine*, Mar. 1990). If the theory is true, the variance of the SNR subjects will equal 0.54, and it will exceed 0.54 if it is false.

a) What null and alternate hypotheses should be used if we are looking for evidence that the theory is not true? What are the type I and type II errors in this case?

b) A sample of size 41 is observed and has an observed variance of 0.5513. What is your conclusion at an α-level of 0.05?

c) What values can we bound the p-value between?

d) What assumptions must be true for this test to be valid? Is the test robust against violations of these assumptions?

And if you have time....

4) In problem 2, we know the probability that we would reject the null hypothesis if it was really true (don't we?). On the other hand it is a little harder to figure out the probability of a type II error.

a) For the hypothesis test in problem 2, determine the rejection region in terms of z for α=0.01.

b) What would this rejection region be in terms of $\hat{p}$? (Hint: Set up the formula and solve for $\hat{p}$.)

c) What would this rejection be in terms of number of sixes observed?

d) What would the "not-rejection" region be in terms of the number of sixes observed?

e) Assume the die had a real p=0.25. Use the normal approximation to the binomial to determine the probability that the result would be in the not-rejection region. This value is called $\beta(0.25)$ = P(Type II error | p=0.25). Notice that there is a different β for every possible value of p!