Statistics 515 - Spring 2001 - Exam 1 Solutions

Part I:

1) P(A)=0.4, P(B)=0.3. If A and B are independent, what is P(A∩B)? **0.3*0.4=0.12**

2) P(A)=0.4, P(B)=0.3. If A and B are mutually exclusive, what is P(A∩B)? **0**

3) A data-entry employee is entering a large list of salaries and one of them is mistyped by either adding or deleting 0s from the end. Is this mistake more likely to affect the mean or the median of the salaries? **mean**

4) A display is being made to compare the populations of the states by using squares. Vermont has a population of approximately 600,000 and is represented by a square that is 1cm by 1cm. How long will each side of the square for West Virginia (population of approximately 1,800,000) be? **Area needs to be $3cm^2$, so sqrt(3) x sqrt(3)**

5) It is often said that a value more than three standard deviations away from the mean is a possible outlier. If the data is approximately normal (or bell-shaped), about what percent of the data will be considered possible outliers?
1-.997 = .003 =0.3%
If the data is skewed, what is the largest percent of the data that could be considered possible outliers?
$1 - (1-1/k^2) = 1/k^2 = 1/3^2 = 1/9 \approx 11.1\%$

6) Let the random variable X have the following distribution:

x	1	2	3
p(x)	0.3	0.4	0.3

What are the mean and variance of X? **mean = 1(0.3) + 2(0.4) + 3(0.3) = 2.0**
variance = $(1-2)^2(0.3) + (2-2)^2(0.4) + (3-2)^2(0.3) = 0.3 + 0 + 0.3 = 0.6$

7) X is a normal random variable with $\mu=5$, $\sigma^2=4$, and $\sigma=2$. Find $P(4 \le X \le 6)$. **= $P((4-5)/2 \le (X-\mu)/\sigma \le (6-5)/2)$**
= $P(-0.5 \le Z \le 0.5) = 2*0.1915 = 0.3830$

8) Z is a standard normal random variable. Find z_0 such that $P(0 \le Z \le z_0) = 0.4821$. **2.10 straight from the table**

Part II:

1) For the data set: 3 feet 1 foot 10 feet 10 feet 1foot
answer the following questions, being **sure to use the appropriate units**. You must show all of your work for credit.

a) Find the mean. **(3'+1'+10'+10'+1')/5 = 25'/5 = 5 feet**

b) Find the median. 1 foot 1 foot **3 feet** 10 feet 10 feet

c) Find the variance. **$((3'-5')^2 + (1'-5')^2 + (10'-5')^2 + (10'-5')^2 + (1'-5')^2)/(5-1)$**
$= (4\ feet^2 + 16\ feet^2 + 25\ feet^2 + 25\ feet^2 + 16\ feet^2)/4 = 86\ feet^2/4 = 21.5\ feet^2$

d) Find the standard deviation. **$sqrt(21.5\ feet^2) \approx 4.637$ feet**

e) Find the mode. **Both 1 foot and 10 feet occur most often.**

2) Based on past censuses and computer models, it is predicted that a large population is 51% female. A survey of 3,000 randomly chosen residents is conducted. Because the population is large and the residents are randomly chosen, this could be considered a binomial experiment.

a) According to the past information, what is the expected number of females in the 3,000 people surveyed?
$\mu=np=3,000(0.51)=1530$

b) According to the past information, what is the variance for the number of females in the 3,000 people surveyed?
$\sigma^2=np(1-p)=3,000(0.51)(1-0.51)=3,000(0.51)(0.49)=749.7$

c) According to the past information, what is the probability that exactly 1,570 of those surveyed will be women? (You do not need to simplify your answer). **(3000 choose 1570) $(0.51)^{1570}$ $(0.49)^{3000-1570}$**

d) Assuming the past information is true, use the central limit theorem to approximate the probability that 1,570 or more of those surveyed will be women.
$P(X \ge 1570) = P(X \ge 1569.5) = P((X-\mu)/\sigma \ge (1569.5-1530)/sqrt(749.7)) = P(Z \ge 1.44) = 0.5 - 0.4251 = 0.0749$