Answers for Suggested STAT 312 Problems for Exam 2

1) 25% of American adults smoke cigarettes. Of these 13% attempted (but failed to quit smoking) during the past year.
A: {An American adult smokes} B: {A smoker attempted to quit smoking last year}
Find P(A), P(B|A), P(A^C), and P(A∩B). State each of these probabilities in the words of the problem.
P(A)=0.25 The probability of a randomly selected American adult smoking is 25%.
P(B|A)=0.13 The probability of a smoking Adult American having tried to quit last year is 13%.
P(A^C)=1-P(A)=1-0.25=0.75 The probability of a randomly selected American adult not smoking is 75%.
P(A∩B)=P(A)P(B|A)=0.25*0.13=0.0325 The probability of a randomly selected American adult being a smoker who tried to quit last year is 3.25%.

2) Pg. 199 #4.7 and 4.9.
4.7: P(A)=0.4 and P(B)=0.3 and A and B are mutually exclusive **so we know P(A|B)=0 and P(B|A)=0.**
a) P(A∩B)=P(B)P(A|B)=0.3*0=0 **b) P(A U B) = P(A) + P(B) - P(A∩B) = 0.4+0.3-0 = 0.7**
c) P(A|B)=0 (they are mutually exclusive) **d) A and B are not independent because P(A|B)≠P(A)**

4.9: P(G)=0.3, P(H)=0.2, and P(G∩H)=0.2
a) P(GUH)=P(G)+P(H)- P(G∩H)=0.3+0.2-0.2=0.3 **b) P(G|H) = P(G∩H)/P(H)=0.2/0.2=1**
c) G and H are not mutually exclusive because P(G|H)≠0
d) G and H are not independent because P(G|H)≠P(G)

3) Consider the partial probability distribution

x	0	1	2	3
p(x)	0.2		0.3	0.2

a) If the only possible values of x are 0, 1, 2, and 3, find P(X=1) **The sum must be 1, so 0.3**
b) Using your answer to a, find E(X) and Var(X)
E(X)=μ=ΣxP(x)=0(0.2) + 1(0.3) + 2(0.3) + 3(0.2) = 0+0.3+0.6+0.6=1.5
Var(X)=σ^2 =Σ(x-μ)^{2}P(x) = (0-1.5)^{2}0.2+(1-1.5)^{2}0.3+(2-1.5)^{2}0.3+(3-1.5)^{2}0.2=1.05
c) Using your answer to a, find **P(X>1) = P(X=1 or X=2) = P(X=1)+P(X=2)-P(X=1 and 2)=0.3+0.2-0=0.5**

4) A group of 15 students needs to be divided into three teams of five students each: the varsity team, the junior varsity team, and the practice squad. How many possible line-ups are possible?
15!/(5!5!5!) =15*14*13*12*11*10*9*8*7*6/(5*4*3*2*1*5*4*3*2*1)= 756,756

5) Pg. 223 #5.49 **(following the box on page 217)**
a) **It is not binomial because the probability of drawing an ace changes after each card is taken from the deck**
b) **It is binomial because each draw from the deck is independent of the previous one (you shuffle), each card is either an ace or not, the probability of getting an ace is always 1/13, and you count up the number of aces**

6) Pg. 225 #5.69a $P(X = 3) = \binom{5}{3} 0.45^3 (1-0.45)^{5-3} = \frac{5!}{3!2!} 0.45^3 0.55^2 = 10(0.91125)(0.3025) = 0.2756531$
b) Find the mean and variance **μ=np=5(.45) = 2.25 σ^2=np(1-p)=5(.45)(.55)=1.2375**
c) Repeat this problem using the normal approximation

$$P(X = 3) = P(2.5 \le X \le 3.5) = P\left(\frac{2.5-2.25}{\sqrt{1.2375}} \le \frac{X-np}{\sqrt{np(1-p)}} \le \frac{3.5-2.25}{\sqrt{1.2375}}\right) \approx P(0.22 \le Z \le 1.12)$$

$$= 0.3686 - 0.0871 = 0.2815$$

7) Pg. 244 # 6.19b **Draw the picture! 0.4904+0.4821=0.9725**

8) Pg. 225 #6.43d **Draw the picture!**

$$P(65 < x < 82) = P\left(\frac{65-60}{10} < \frac{x-\mu}{\sigma} < \frac{82-60}{10}\right) = P(0.5 < z < 2.2) = 0.4861 - 0.1915 = 0.2946$$

9) Pg. 280 #7.7a-b

Sample	**Mean**	**Sample**	**Mean**	**Sample**	**Mean**	**Sample**	**Mean**	**Sample**	**Mean**	**all have probability 1/5*1/5=1/25**
1,1	**1**	**3,1**	**2**	**5,1**	**3**	**7,1**	**4**	**9,1**	**5**	
1,3	**2**	**3,3**	**3**	**5,3**	**4**	**7,3**	**5**	**9,3**	**6**	
1,5	**3**	**3,5**	**4**	**5,5**	**5**	**7,5**	**6**	**9,5**	**7**	
1,7	**4**	**3,7**	**5**	**5,7**	**6**	**7,7**	**7**	**9,7**	**8**	
1,9	**5**	**3,9**	**6**	**5,9**	**7**	**7,9**	**8**	**9,9**	**9**	

$\bar{x}$	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
P($\bar{x}$ **)**	**1/25**	**2/25**	**3/25**	**4/25**	**5/25**	**4/25**	**3/25**	**2/25**	**1/25**

10) Pg. 403 #9.89 $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} \Rightarrow 1550 \pm 2.80 \frac{125}{\sqrt{25}} \Rightarrow 1550 \pm 70 \Rightarrow (\$1480,\$1620)$

11) Pg. 397 #9.63 and verify that n is large enough to trust the result

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \Rightarrow 0.40 \pm 1.65 \sqrt{\frac{0.4(1-0.4)}{1000}} \Rightarrow 0.40 \pm 0.026 \Rightarrow (0.374, 0.426)$$

$n\hat{p} = 1000(0.4) = 400$ and $n(1-\hat{p}) = 1000(1-0.4) = 600$ are both 5 or larger so n is large enough

12) How large of a sample size is needed to make a 95% confidence interval for proportions that is ±.015 (1.5%)

Recall that the worst possible case is p=0.5, so

$$\pm 0.015 = \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \Rightarrow 0.015 = 1.96 \sqrt{\frac{0.5(1-0.5)}{n}} \Rightarrow n = \frac{1.96^2 (0.5)(0.5)}{0.015^2} \Rightarrow n = 4268.444$$

so we need to have a sample of at least size 4269