Score Vectors and Information Matrices for EM example

$$S(y_i, \mathbf{z_i}, \psi) = \begin{bmatrix} z_{i1}(y_i - \mu_1) \\ \vdots \\ z_{i1}(y_i - \mu_T) \\ \frac{z_{i1}}{\pi_1} - \frac{z_{iT}}{\pi_T} \\ \vdots \\ \frac{z_{i,T-1}}{\pi_{T-1}} - \frac{z_{iT}}{\pi_T} \end{bmatrix}$$

$$S^*(y_i, \psi) = E[S(y_i, \mathbf{z_i}, \psi) \mid \mathbf{y_i}] = \begin{bmatrix} \pi_{i1}(y_i - \mu_1) \\ \vdots \\ \pi_{i1}(y_i - \mu_T) \\ \frac{\pi_{i1}}{\pi_1} - \frac{\pi_{iT}}{\pi_T} \\ \vdots \\ \frac{\pi_{i,T-1}}{\pi_{T-1}} - \frac{\pi_{iT}}{\pi_T} \end{bmatrix}$$

$$B(y_i, \mathbf{z_i}, \psi) = \begin{bmatrix} z_{i1} & 0 & \dots & \dots & \dots & 0 \\ 0 & \ddots & \ddots & & & \vdots \\ \vdots & \ddots & z_{i,T} & \ddots & & \vdots \\ \vdots & & \ddots & \frac{z_{i1}}{\pi_1^2} + \frac{z_{iT}}{\pi_T^2} & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & \dots & \dots & 0 & \frac{z_{i,T-1}}{\pi_{T-1}^2} + \frac{z_{iT}}{\pi_T^2} \end{bmatrix}$$

$$B^*(y_i, \psi) = \begin{bmatrix} \pi_{i1} & 0 & \dots & \dots & \dots & 0 \\ 0 & \ddots & \ddots & & & \vdots \\ \vdots & \ddots & \pi_{i,T} & \ddots & & \vdots \\ \vdots & & \ddots & \frac{\pi_{i1}}{\pi_1^2} + \frac{\pi_{iT}}{\pi_T^2} & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & \dots & \dots & 0 & \frac{\pi_{i,T-1}}{\pi_{T-1}^2} + \frac{\pi_{iT}}{\pi_T^2} \end{bmatrix}$$

$$E\left[S(y_i, \mathbf{z_i}, \psi)\mathbf{S}'(\mathbf{y_i}, \mathbf{z_i}, \psi)\right] = \begin{bmatrix} & D_1 & \begin{matrix} D_2 \\ \nabla'_{T-1} \end{matrix} \\ D_2 & \nabla_{T-1} & D_3 \end{bmatrix}$$

$$D_{1tt} = \pi_{it}(y_i - \mu_t)^2,\ D_{2tt} = \frac{\pi_{it}(y_i - \mu_t)}{\pi_t},\ D_{3tt} = \frac{\pi_{it}}{\pi_t} - \frac{\pi_{it}}{\pi_t},\ \nabla'_{T-1} = \left(-\frac{\pi_{iT}}{\pi_T}(y_i - \mu_T), \dots, -\frac{\pi_{iT}}{\pi_T}(y_i - \mu_T)\right)$$