Midterm

1. Consider the following 2^3 replicated (n=2) factorial design:

Run	Response
(1)	121.6, 121.4
a	129.3, 127.3
b	104.8, 107.4
ab	144.7, 143.5
c	126.7, 123.4
ac	129.4, 126.0
bc	111.1, 108.2
abc	142.5, 146.4

   (a) Compute the MSPE for this experiment; you will use it as an estimate of σ for sample size calculations. Assume $\alpha = .05$ for the following questions.

   (b) Suppose we wished to detect an A main effect of size -2 with 80% power. How large a sample size would we need?

   (c) Suppose we wished to detect an AB interaction effect of size 1.5 with 90% power. How large a sample size would we need?

   (d) Researchers sometimes like to compute posterior, or observed, power. Estimate the A and B main effects from the above experiment, and then use these estimates to compute the observed power for detecting A and B effects of these magnitudes.

2. Consider a CRD on the quantitative factor X with levels 24, 28, 32, and 36. We are interested in the quadratic contrast $L_Q = c_1 E(Y|X = 24) + \ldots + c_4 E(Y|X = 36)$ with contrast coefficients $\mathbf{c}' = (\mathbf{1}, -\mathbf{1}, -\mathbf{1}, \mathbf{1})$.

   (a) Using the quadratic model $E(Y|X) = \beta_0 + \beta_1 X + \beta_2 X^2$, write L_Q as a function of β_2. Argue that a test of $L_Q = 0$ is equivalent to a test of $\beta_2 = 0$.

   (b) Can you make the same argument about L_Q for the cubic model $E(Y|X) = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 X^3$? What conclusions can you reach about the test for a quadratic effect?

3. Suppose we have a 3 by 3 table with the following set of observed sample responses.

		B	
A	12,3	17	
	10,11	3	
			19, 16, 17

(a) For the additive model, are the population marginal means for A and B estimable?

(b) Add the observations 6 and 12 to the (2,3) cell. Are the population marginal means for A and B estimable? Briefly explain any differences between your two results.

(c) Using the augmented data set, compare and contrast the Type III and Type IV hypotheses for the interaction model for A, B and AB.