

## Midterm

1. Consider a CRD on the quantitative factor  $X$  with levels 60, 70, 80, 90 and 100. We are interested in the linear contrast  $L = c_1 E(Y|X = 60) + \dots + c_5 E(Y|X = 100)$  with contrast coefficients  $\mathbf{c}' = (-4, -2, 0, 2, 4)$ .
  - (a) Under the linear model  $E(Y|X) = \beta_0 + \beta_1 X$ , show that a test of  $L = 0$  is equivalent to a test of  $\beta_1 = 0$ .
  - (b) Analyze the data in the pilot study below. Is the linear contrast  $L$  significant? Report on lack of fit as well.
  - (c) Write the non-centrality parameter for the contrast  $L$  as a function of  $\beta_1$ . Assuming  $\sigma = 10$ , construct a power curve (as a function of  $n$ ) for  $\beta_1 = .5$ . How many replications would we need to detect  $\beta_1 = .5$  with 80% power?

| X   | Y                |
|-----|------------------|
| 60  | 27.2, 23.9, 18.5 |
| 70  | 32.3, 29.7, 1.3  |
| 80  | 40.2, 37.8, 19.6 |
| 90  | 19.4, 37.1, 32.7 |
| 100 | 36.0, 29.0, 37.1 |

2. Consider the following  $2^4$  unreplicated factorial design:

| Run | Response | Run  | Response |
|-----|----------|------|----------|
| (1) | 7.2      | d    | 12.9     |
| a   | 6.5      | ad   | 19.4     |
| b   | 6.2      | bd   | 12.4     |
| ab  | 4.6      | abd  | 17.5     |
| c   | 9.5      | cd   | 16.2     |
| ac  | 4.1      | acd  | 18.9     |
| bc  | 5.8      | bcd  | 13.0     |
| abc | 11.4     | abcd | 22.0     |

- (a) Which effects does Lenth's test identify as significant?
- (b) Suppose we assumed 3-way and 4-way interactions are not significant and excluded them from the model, so that the model is no longer saturated. Using F tests (or t tests), which effects are now significant?
- (c) Suppose we assumed all non-significant terms from Part 1 should be excluded from the model; which effects are significant?
- (d) Compare all three approaches. What are the advantages and disadvantages of each?

3. Suppose we have two two-way tables with the following distributions of 15 experimental units among the sample cells. You will want to create some simulated data and then run appropriate models in SAS in order to answer the following questions.

|   |   |   |
|---|---|---|
|   | B |   |
|   | 3 | 3 |
| A | 3 | 3 |
|   | 3 | 0 |

|   |   |   |
|---|---|---|
|   | B |   |
|   | 1 | 4 |
| A | 6 | 1 |
|   | 2 | 1 |

- (a) What would be the Type III contrasts (interaction model) for factors A and B in the first table?
- (b) What would be the Type I contrasts for factors A and B (interaction model) in the second table? Confirm that your contrasts would test Yandell's hypotheses on page 170. Yandell felt that the test on B was "rather bizarre"—would you agree with this statement in practice?
- (c) Comment on the contrasts you have constructed and the implications in terms of inference on badly-balanced, but complete designs versus well-balanced, but incomplete designs.