STAT718A SHAPE AND IMAGE ANALYSIS Spring Session 2009

Mid-term examination (take home)

Date given out: 3/18/09, 4pm. Please submit your solutions to the problems to me by 4pm March 25, 2009.
This midterm examination contributes 25% of the course grade. Please attempt all questions on your own and do not confer with others. There are FOUR questions in total, each worth equal weight. You may use R to help in answering the questions, and please provide your R commands.

1. (a) Define what is meant by the shape of an object.
   (b) Define what is meant by the size-and-shape of an object.
   (c) Define what is meant by a size measure.
   (d) Show that the centroid size is a size measure.
   (e) Define the Bookstein shape co-ordinates, and provide one advantage and one disadvantage of using these shape co-ordinates.
   (f) Define the full Procrustes distance and the partial Procrustes distance, and state the range of these distances.
   (g) Define the partial Procrustes tangent co-ordinates and the full Procrustes tangent co-ordinates.
   (h) Provide a reason why Bookstein's shape variables could be preferred to the internal angles as shape co-ordinates.

2. Consider the following three triangles:

$$A : (1,1), (4,1), (1,4).$$

$$B : (10,0), (14,4), (12,4).$$

$$C : (3,5), (3,8), (-1,5).$$

   (a) Obtain the Bookstein shape co-ordinates and Kendall shape co-ordinates of the triangles, where the baseline is taken as points 1 and 2.
   (b) Rank the triangles in order in terms of centroid size, area, baseline size.
   (c) Rank the triangles in terms of closest distance to an equilateral triangle. Explain your reasoning.
   (d) Obtain Kendall's spherical co-ordinates for each of the triangles.
   (e) Which of the triangles is closest to the set of collinear triangles? Explain your reasoning.

3. (a) Consider two centered planar configurations $y = (y_1, \ldots, y_k)^T$ and $w = (w_1, \ldots, w_k)^T$, where $y_i, w_i, i = 1, \ldots, k$ are complex numbers. Derive the expression for the full Procrustes fit w^P of w onto y.

   (b) Consider the case where

$$y = (-1/2 - i\sqrt{3}/6, 1/2 - i\sqrt{3}/6, i2\sqrt{3}/6)^T$$

   is the anti-clockwise labelled equilateral triangle of unit centroid size and

$$w = (-1/2 + i\sqrt{3}/6, 1/2 + i\sqrt{3}/6, -i2\sqrt{3}/6)^T$$

   is the clockwise labelled equilateral triangle of unit centroid size. Use your answer to part (a) to obtain the full Procrustes fit of w onto y in this example, and check your result using `procOPA()` in R.

4. On the course webpage under the heading 'Mid-term' you will find several datasets. Please carry out statistical shape analysis of the dataset which has been assigned to you (see below). Your findings should be presented as a short report, including any relevant plots.

   To read in the data you can use the command:

   ```
   x <- read.in( "filename" , k=10, m=2)
   ```

   You should have $n = 30$ configurations with $k = 10$ landmarks in $m = 2$ dimensions.

   I suggest that you carry out Procrustes analysis of the data, including computation of the mean shape and the principal components. Investigate the relationship between size, Riemannian distance and the first few PC scores. Consider the percentage of variability explained by the first few PCs, and describe geometrically the structure of the PCs. Depending on what you see you may then need to do further analysis...

Dataset assignments

data1.txt: Bonnie Coggins
data2.txt: Pahal Dalal
data3.txt: Blake Hill
data4.txt: Tomonori Ishikawa
data5.txt: Nicole Lewis
data6.txt: Crystal Mack
data7.txt: Chris McMahan
data8.txt: Safiya Moran
data9.txt: Kenton Oliver