STAT718A   SHAPE AND IMAGE ANALYSIS   Spring Session 2009

Homework 5

Date given out: 4/13/09. Please submit your solutions to the problems to me by 4pm April 20, 2009. This homework contributes 10% of the course grade.

1. Show that the following two forms of the conditional probability $P(S_{ij}|\ rest)$ for the Ising model for the 4-neighbourhood graph are equivalent and make the connection between the parameterisations:

$$P(S_{ij} = 1|rest) = e^T/(1 + e^T)$$

   where $T = \beta'(\sum_{n'bors} S_{i'j'}) - 2\beta'$ and

$$P(S_{ij} = 1|rest) = e^{-\beta n_0}/(e^{-\beta n_1} + e^{-\beta n_0})$$

   where n_0 is the number of black neighbors and n_1 is the number of white neighbors.

2. For the Ising model with four neighbors evaluate the conditional probability that a pixel is white given that there are 3 white neighbors, when $\beta = 2, 1, 0, -1, -2$. Give an interpretation to β.

3. For the Ising model with four neighbors, draw 5 graphs, one corresponding to each of $0, 1, 2, 3, 4$ black neighbors. On each graph sketch the probability (as a function of β) of flipping the central pixel to black for both the Metropolis algorithm and the Gibbs' sampler. Draw these over the range $0 \le \beta \le 2$.

4. (a) Provide the maximum likelihood estimates and standard errors under the offset isotropic Gaussian model for the male and female Chimpanzee datasets (panm.dat and panf.dat) in the shapes package. You should use the updated version of isomle which you can find on the course web site in the file isomle.txt.

   (b) Provide a plot of the MLE mean shapes, and comment.

   (c) Carry out a likelihood ratio test to examine whether or not the male and females have the same shape distribution parameters.