STAT718A SHAPE AND IMAGE ANALYSIS Spring Session 2009

Homework 4

Date given out: 3/30/09. Please submit your solutions to the problems to me by 4pm April 8, 2009. This homework contributes 10% of the course grade.

1. Consider an image where the pixels belong to two classes, D for the dark class (e.g. background) and B for the bright class (e.g. object). Consider the continuous distribution of grey levels to be a mixture of two normal distributions $N(\mu_B, \sigma_B^2)$ and $N(\mu_D, \sigma_D^2)$ with mixture proportions π_B and $\pi_D = 1 - \pi_B$. Derive an expression for the automatic threshold t which minimizes the error:

   $$L = \pi_B P(X \leq t | X \sim N(\mu_B, \sigma_B^2)) + \pi_D P(X > t | X \sim N(\mu_D, \sigma_D^2)),$$

   where $t \in \mathbb{R}$.

   Simplify your expression for $\mu_D = 0, \mu_B = 1, \sigma_D = \sigma_B = 1$.

   Simplify further for $\pi_D = \pi_B = 1/2$.

   [Hint: To minimize L differentiate with respect to t, and solve $dL/dt = 0$. If $F(t)$ is the c.d.f. of distribution, then $dF(t)/dt = f(t)$, the density of the distribution.]

2. A 64×64 image with a total of four distinct grey levels $(0, 1, 2, 3)$ has grey level frequency distribution:

grey level	frequency
0	1456
1	683
2	789
3	1168
total	4096

   It is of interest to threshold the image into two classes. What is the choice of threshold that minimizes the within-group variance?

   What is the choice of threshold that minimizes the Kullback-Leibler divergence where $\mu_1 = 0.5; \mu_2 = 2.5; \sigma_1 = 0.5 = \sigma_2$.

3. Suppose that a true binary scene S_{ij} has been observed with noise so that

   $$x_{ij} = S_{ij} + \epsilon_{ij}$$

   where $\epsilon_{ij} \sim N(0, \sigma^2)$ independently. To reconstruct the scene we threshold the image to give a binary image y_{ij} by allocating $y_{ij} = 1$ if $x_{ij} > 1/2$ and $y_{ij} = 0$ otherwise.

   The error rate E is defined as the probability of mis-classification. Here we have

$$E = P(x_{ij} > 1/2 \ \text{AND} \ S_{ij} = 0) + P(x_{ij} \le 1/2 \ \text{AND} \ S_{ij} = 1).$$

Show that

$$E = 1 - \Phi(\frac{1}{2\sigma}).$$

Obtain the approximate error rate after applying the 5×5 box filter to an image containing large objects.

4. [*Optional] By fitting a quadratic surface (by least squares) of the form

$$x_{ij} = a_1 + a_2 i + a_3 j + a_4 i^2 + a_5 ij + a_6 j^2 + \varepsilon_{ij}$$

to a 5×5 neighbourhood, verify that the filter is given by the weights:

$$w_{ij} = \frac{1}{25}(1 + \frac{10 - 5i^2}{7} + \frac{10 - 5j^2}{7}) \qquad j = -2, -1, 0, 1, 2, \quad i = -2, -1, 0, 1, 2$$

5. Using the binary image A and structuring element K below, display the resulting output image for each of the following:
(a) $A \oplus K$, (b) $A \ominus K$, (c) $A \circ K$, (d) $A \bullet K$, (e) $(A \ominus K)^c$, (f) $A^c \oplus \check{K}$.

Check whether (e) equals (f), and explain why this is to be expected.

$A =$

	•	•	•	•		
	•	+		•	•	
	•	•	•	•	•	
	•	•		•	•	
		•	•	•	•	

$K =$

•		
•	+	
•	•	•

[For each image the origin is denoted by $+$]