STAT718A SHAPE AND IMAGE ANALYSIS Spring Session 2009

Homework 2

Date given out: 2/9/09. Please submit your solutions to the problems to me by 5.15pm February 18, 2009. This homework contributes 10% of the course grade.

1. (a) For the female gorilla data (see Section 1.2.2 of the notes) find the centroid sizes of the gorilla landmarks and provide a plot of the centroid size versus the Riemannian distance ρ of each individual to the mean shape. The landmark data can be found in the shapes package in `gorf.dat`, and you should use R for your calculations.

   (b) Obtain Bookstein's shape variables for the specimens and provide a scatter plot of the data, registered on the baseline 1,2. Obtain the Bookstein mean shape, and add it to your plot.

   (c) Obtain the mean shape using full Procrustes analysis. Add this to your plot as well. How far apart in terms of Riemannian shape distance ρ (command `riemdist`) is the Procrustes mean from the Bookstein mean?

   (d) Write down the Bookstein's shape variables for the largest specimen using landmarks 1, 2 as the baseline.

   (e) Find the specimen which is the furthest away from the mean in terms of Riemannian distance.

2. (a) Find the full Procrustes mean shapes of the the male (gorm.dat) and female (gorf.dat) gorillas separately by using the `procGPA` command with option eigen2d=TRUE (uses complex eigenvectors) and eigen2d=FALSE (uses the iterative GPA method). Check whether they give the same result (up to an arbitrary scale and rotation). Compare the percentages of variability explained by the first three PCs using both routines.

   (b) Which of the two methods of calculating the Procrustes mean shape is the fastest? How much faster is it?

   Hint : The following might be useful:

```
date()
for (i in 1:100){
ans <- procGPA( gorf.dat ,eigen2d=FALSE )
}
date()
```

3. Give an educated guess as to what the full Procrustes mean shape would be from a dataset of two equilateral triangles - one clockwise and the other anti-clockwise labelled. Find the full Procrustes mean shape using the routine `procGPA`.

4. Consider a pre-shape $z = x + iy$ ($k-1-$vector). Obtain the pre-shapes which are the furthest possible Riemannian distance from this pre-shape.

5. Obtain an expression for the full Procrustes distance d_F in terms of the partial Procrustes distance d_P. Obtain a Taylor series expansion of d_F in terms of d_P giving the first two non-zero terms explicitly.

6. Plot the full Procrustes and partial Procrustes distances versus ρ over the range of ρ. Discuss the relative values of the distances over the range of ρ.