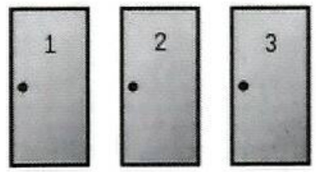
Name: ______________________________

Standard 12: Quiz Six – Basic Probability Results

Let's say you're on the famous game show, Let's Make a Deal and you were lucky enough to be selected to play for the big deal of the day! The setup of the game is this – there are three doors and there is a very expensive prize behind one of the doors, like a new car; the other two doors yield you nothing!



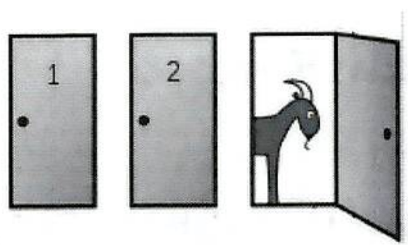
You have three choices to choose from, let A denote the event that you choose the winning door. Find the following probabilities for **A = You chose the winning door:**

a. $P(A) = 1/3 = .\overline{33}$

b. $P(A^c) = 1 - \frac{1}{3} = 2/3 = .\overline{66}$

Standard 13: Quiz Six – Probability – Conditional Probability

Now, regardless if your initial guess was correct the host will show you that there is a goat behind one of the other doors. For example, if you chose door one the host might show you door three which tells you that the attractive prize is definitely behind door 1 or door 2.



Now, he will ask you if you want to change your mind – you now have two options: you can stick with your initial gut instinct or you can choose to switch to the other door. Let B denote the event that you choose the winning door this time. Find the following probabilities for **B = You chose the winning door and C = door 3 is a loser**

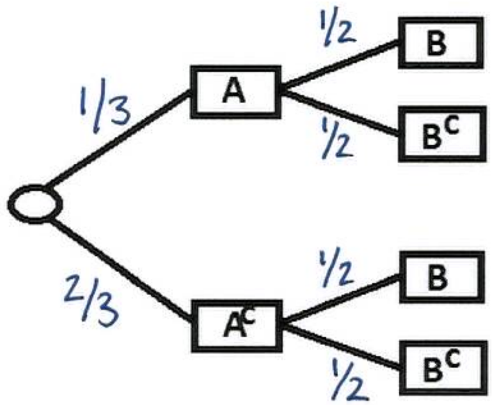
$P(B|C) = 1/2 = .5$

$P(B^c|C) = 1 - 1/2 = 1/2 = .5$

Name: ____________________

Standard 14: Quiz Six – Probability – Disjoint vs. Independent

a) Use standards 12 and 13 to assign the correct probability to the correct branch on the tree diagram. Note: you need to assume **independence** of events A and B, your 1st and 2nd choice.



b) Find the following probabilities and choose the best strategy for winning the big deal of the day, i.e. is it better to assume your first answer was wrong and switch or assume your first answer was right and keep it?

$P(A \cap B) = P(A) \cdot P(B|A) \overset{\text{ind}}{=} P(A)P(B) = 1/3 \cdot 1/2 = 1/6$

$P(A^C \cap B) = P(A^c) \cdot P(B|A^c) \overset{\text{ind}}{=} P(A^c)P(B) = 2/3 \cdot 1/2 = 1/3$

It is better to assume we were wrong on our first guess, because the probability of winning is higher.

Name: ______________________________

Standard 17: Quiz Six – Counting Techniques

Domino's Pizza sells two topping medium pizza's for $5.99. (HOW CAN THEY AFFORD TO DO IT?) Domino's offers three types of crust, six types of sauce, and twenty four toppings.

How many unique order combinations of a Domino's two topping pizza are there? Remember you have to choose the crust, the sauce and two of the 24 toppings – (multiply the number of possibilities for each choice to get your final answer.) You only need to write out the formula for this one.

# of crusts # of sauces # of 2-topping combos

$$3 \times 6 \times {}_{24}C_2$$

$$= 3 \times 6 \times \frac{24!}{2!(24-2)!}$$

$$= 3 \times 6 \times \frac{24!}{2!\,22!}$$

$$= 3 \times 6 \times \frac{24 \times 23}{2}$$

$$= 3 \times 6 \times 276$$

$$= 4{,}968$$

You can order a pizza every day for almost 14 years without having the exact same pizza!

Name: ______________________________

Standard 18: Quiz Six – Probability Dist. – Discrete Distribution

Consider the following discrete distribution for the number books read over break.

Number of Books	P(x)	x*P(x)
0	.60	0
1	.23	.23
2	.07	.14
3	.07	.21
4	.02	.08
5	.01	.05

Provide the three assumptions needed to show this is a discrete distribution.

1. Countable X

2. $0 \leq P(x) \leq 1$

3. $\sum P(x) = 1$

Find the mean of this distribution. (Hint: fill in the column in the table above)

$$E(x) = \sum x P(x) = 0 + .23 + .14 + .21 + .08 + .05 = .71$$

Sadly the mean is less than one book!

Name: ______________________________

Standard 19: Quiz Six – Probability Distributions– Binomial

1) A recent poll by the South Carolina House Republican Caucus showed that 34.5 percent of likely republican primary voters in South Carolina prefer Donald Trump to the rest of the presidential candidates.
Let's consider the case where we want to confirm or deny the accuracy of this claim and instead of having lecture I made all 72 students in class call random phone numbers with a South Carolina area code until we were able to ask one hundred random likely republican primary voters in South Carolina whether they prefer Donald Trump or another candidate for president.

   a. Can we use the binomial here? Consider the three requirements – which are met or violated here and why?

   1) Fixed $n=100$

   2) Independent trials

   3) Identical trials

   $X_n = \#$ in 100 calls that prefer Donald Trump

   $p=.345$

   b. Regardless of your answer in a, find the mean and standard deviation of the binomial distribution.

   $$\mu = np = 100 \cdot .345 = 34.5$$

   $$\sigma = \sqrt{npq} = \sqrt{100 \cdot .345 \cdot (1-.345)} = \sqrt{22.5975} = 4.753683$$

   c. If twenty of one hundred interviews were with a likely republican primary voters in South Carolina that prefers Donald Trump would this be much different than what we would expect? If so, does this result suggest that Donald Trump support is less than or greater than 34.5%?

   Yes, this would be strange because $X=20$ is more than 3 standard deviations below the mean.

Name: ______________________________

Standard 20: Quiz Six – Probability Dist. – Bell-Shaped Distributions Z-Scores

Suppose the average height of an American male is normally distributed with a mean of five foot - ten inches tall, which is 69.7 inches, and a standard deviation of three inches.

Recall: $z = \frac{observation - mean}{standard\ deviation} = \frac{x-\mu}{\sigma}$

a. Yao Ming is seven foot –six inches, which is 90 inches tall – is he outlying?

$z = \frac{90-69.7}{3} = 6.7\overline{6} > 3$ so he is outlyingly tall.

b. Verne Troyer is two foot –eight inches, which is 32 inches tall – is he outlying?

$z = \frac{32-69.7}{3} = -12.5\overline{6} < -3$ so he is outlyingly short.

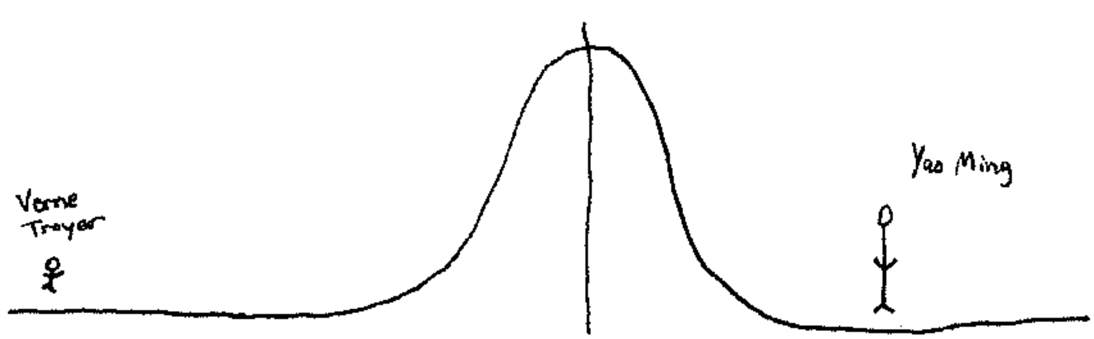
c. Donald Trump is six foot –two inches, which is 74 inches tall – is he outlying?

$z = \frac{74-69.7}{3} = 1.4\overline{3}$

$-3 < 1.4\overline{3} < 3$ so he is not outlying.

d. Among these three people who is most outlying? How can you justify this answer?

Verne Troyer is most outlying since his z-score has the highest magnitude – meaning he is further from the mean.

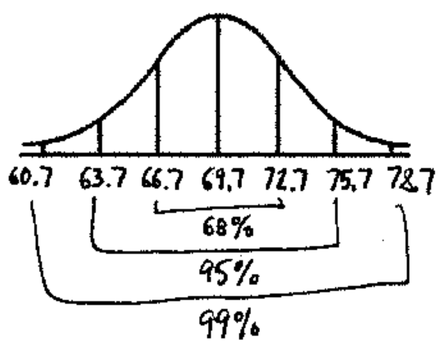


Name: ______________________________

Standard 21: Quiz Six – Probability Dist.– Bell-Shaped Distributions finding Probabilities

Again, suppose the average height of an American male is normally distributed with a mean of five foot - ten inches tall, which is 69.7 inches, and a standard deviation of three inches.

a. According to the empirical rule 95% of the observations lay between which two values?



95% of observations lay between 63.7 inches and 75.7 inches.

b. Using your answer in part a, calculate the proportion of observations between those two values using the z-table approach which is more accurate. How good is the empirical rule as an approximation, that is how close is this to 95%?

$$P(63.7 < X < 75.7) = P(X < 75.7) - P(X < 63.7)$$
$$= P\left(Z < \frac{75.7-69.7}{3}\right) - P\left(Z < \frac{63.7-69.7}{3}\right)$$
$$= P(Z < 2) - P(Z < -2)$$
$$= .9772 - .0228$$
$$= .9544$$

← This is close, but not exactly 95%