$\boxed{\text{The Normal Distribution}}$ $N(\mu, \sigma^2)$

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} I(x \in \mathbb{R}) I(0 \le \sigma^2 < \infty) I(\mu \in \mathbb{R})$$

Relationships

① $X \sim N(0,1) \rightarrow X^2 \sim \chi^2(1)$

$Y = g(x) = x^2$

$Y^{-1} = g^{-1}(y) = \pm\sqrt{y}$

$A_0 = \{0\}$

$A_1 = (\infty, 0)$ $\quad g_1(x) = x^2 \quad g_1^{-1}(y) = -\sqrt{y} \quad \frac{d}{dy} g_1^{-1}(y) = \frac{-1}{2\sqrt{y}}$

$A_2 = (0, \infty)$ $\quad g_2(x) = x^2 \quad g_2^{-1}(y) = \sqrt{y} \quad \frac{d}{dy} g_2^{-1}(y) = \frac{1}{2\sqrt{y}}$

$$f_y = f_X(-\sqrt{y}) \left|\frac{-1}{2\sqrt{y}}\right| + f_X(\sqrt{y}) \left|\frac{1}{2\sqrt{y}}\right| = \frac{1}{2\sqrt{2\pi y}} e^{-y/2} + \frac{1}{2\sqrt{2\pi y}} e^{-y/2}$$

$$= \frac{1}{\sqrt{2\pi y}} e^{-y/2}$$

$$= \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\sqrt{y}} e^{-y/2}$$

$$= \frac{1}{\Gamma(\frac{1}{2})} \cdot y^{1/2 - 1} e^{-y/2} \rightarrow \chi^2_{(1)}$$

② $X \sim N(\mu, \sigma^2) \longrightarrow Z = \frac{X-\mu}{\sigma} \sim N(0,1)$

③ $X_1, X_2, X_3 \ldots X_n \overset{ind}{\sim} N(\mu_i, \sigma_i^2) \longrightarrow \Sigma a_i Y_i \sim N(\Sigma a_i \mu_i, \Sigma a_i^2 \sigma_i^2)$

④ $X_1, X_2, X_3 \ldots X_n \overset{iid}{\sim} N(\mu, \sigma^2) \longrightarrow \Sigma a_i Y_i \sim N(\mu \Sigma a_i, \sigma^2 \Sigma a_i^2)$

⑤ $X_1, X_2, X_3 \ldots X_n \overset{iid}{\sim} N(\mu, \sigma^2) \longrightarrow \frac{1}{n} \Sigma Y_i = \bar{Y} \sim N(\mu, \sigma^2/n)$

⑥ $X_1, X_2, X_3 \ldots X_n \overset{iid}{\sim} N(\mu, \sigma^2) \longrightarrow \Sigma Y_i \sim N(n\mu, n\sigma^2)$

⑦ $X_i \sim N(0,1) \rightarrow X_i^2 \sim \chi^2_{(1)}$

⑧ $X_1, X_2, \ldots X_n \overset{iid}{\sim} N(0,1) \longrightarrow \Sigma X_i^2 \sim \chi^2_{(n)}$

⑨ $X_1, X_2 \overset{iid}{\sim} N(0,1) \longrightarrow X_1/X_2 \sim$ Cauchy

⑩ $X_1, X_2, \ldots X_n \overset{iid}{\sim} N(\mu, \sigma^2) \longrightarrow \frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{(n-1)}$

⑪ $X \sim N(0,1) \perp\!\!\!\perp W \sim \chi^2(v) \longrightarrow \frac{Z}{\sqrt{W/v}} \sim t_{(v)}$

Exponential Family

$$f_X(x) = \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\sigma} \cdot e^{-\frac{(x^2 - 2x\mu + \mu^2)}{2\sigma^2}} = \underbrace{\frac{1}{\sqrt{2\pi}}}_{h(x)} \cdot \underbrace{\frac{1}{\sigma} e^{-\frac{\mu^2}{2\sigma^2}}}_{c(\sigma,\mu)} e^{x^2\left(\frac{-1}{2\sigma^2}\right) + x\left(\frac{\mu}{\sigma^2}\right)}$$

$h(x)$, $c(\sigma,\mu)$, $t_1(x) = x^2$, $t_2(x) = x$

$w_1(\sigma,\mu) = \frac{-1}{2\sigma^2}$ $\quad w_2(\sigma,\mu) = \frac{\mu}{\sigma^2}$

MGF

$$M_x(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx} \left[\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}\right] dx$$

let $z = \frac{x-\mu}{\sigma}$

$$= \int_{-\infty}^{\infty} e^{\sigma z t + \mu t} \left[\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\right] dz$$

$$= e^{\mu t} \int_{-\infty}^{\infty} \left(e^{\sigma z t} \left[\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\right]\right) dz$$

$$= e^{\mu t} M_z(\sigma t)$$

$$M_z(t) = \int_{-\infty}^{\infty} e^{tz} \left[\frac{1}{\sqrt{2\pi}} e^{-z^2/2}\right] dz = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{tz - z^2/2} dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{tz - z^2/2 + t^2/2 - t^2/2} dz = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{1}{2}t^2} e^{\frac{-1}{2}(z^2 - 2tz + t^2)} dz$$

$$= e^{\frac{1}{2}t^2} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-1}{2}(z-t)^2} dz$$

$$= e^{\frac{1}{2}t^2}$$

$$= e^{\mu t}\left[e^{\frac{1}{2}t^2}\right] = e^{t\left(\mu + \frac{t^2}{2}\right)}$$

Expected Value and Variance

$$E(x) = \int_{-\infty}^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx$$

let $w = (x-\mu)^2$ $\quad dw = 2(x-\mu)dx$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (x-\mu) e^{\frac{-(x-\mu)^2}{2\sigma^2}} + \frac{\mu}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$$= \underbrace{\frac{1}{2\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-w/2\sigma^2}} + \mu \int_{-\infty}^{\infty} f_x(x)\,dx$$

= this doesn't close but it should go to zero $+ \mu$

$$E(x) = \frac{d}{dt} M_x(t)\Big|_{t=0}$$

$$= d/dt\ e^{t(\mu + \frac{\sigma^2 t}{2})} = d/dt\ e^{t\mu} e^{\frac{\sigma^2 t^2}{2}} \qquad u = e^{tm} \quad v = e^{\frac{\sigma^2 t^2}{2}}$$

$$= \sigma^2 t\, e^{t(\mu + \frac{\sigma^2 t}{2})} + \mu\, e^{t(\mu + \frac{\sigma^2 t}{2})} \qquad du = \mu e^{t\mu} \quad dv = \sigma^2 t\, e^{\frac{\sigma^2 t^2}{2}}$$

$$= e^{t(\mu + \frac{\sigma^2 t}{2})} \left[\sigma^2 t + \mu\right]\Big|_{t=0}$$

$$= 1\,[0 + \mu]$$

$$= \mu$$

$$E(x^2) = \frac{d^2}{dt^2} M_x(t)\Big|_{t=0} = \frac{d}{dt}\, e^{t(\mu + \frac{\sigma^2 t}{2})} \left[\sigma^2 t + \mu\right] \qquad u = e^{t(\mu + \frac{\sigma^2 t}{2})} \quad v = \sigma^2 t + \mu$$

$$= \sigma^2 e^{t(\mu + \frac{\sigma^2 t}{2})} + (\sigma^2 t + \mu)^2 e^{t(\mu + \frac{\sigma^2 t}{2})}\Big|_{t=0} \qquad du = e^{t(\mu + \frac{\sigma^2 t}{2})}\left[\sigma^2 t + \mu\right] \quad dv = \sigma^2$$

$$= \sigma^2 e^0 + \mu^2 e^0$$

$$= \sigma^2 + \mu^2$$

$$Var(x) = E(x^2) - (E(x))^2 = \left[\sigma^2 + \mu^2\right] - \mu^2$$

$$= \sigma^2$$

Sufficient Statistic

$$f(\underline{x}|\mu) = \prod f_X(x) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{\frac{-1}{2\sigma^2}\sum(x_i-\mu)^2}$$

σ^2 known

$$= \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{\frac{-1}{2\sigma^2}\sum(x_i-\bar{x}+\bar{x}-\mu)^2}$$

$$= \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{\frac{-1}{2\sigma^2}(\sum_1^n x_i-\bar{x})^2} \; e^{\frac{-n}{2\sigma^2}(\bar{x}-\mu)^2}$$

$$= \underbrace{\left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{\frac{-n}{2\sigma^2}(\bar{x}-\mu)^2}}_{g(t|\mu)} \; \underbrace{e^{\frac{-1}{2\sigma^2}(\sum_1^n x_i-\bar{x})}}_{h(\underline{x})}$$

$\therefore$ $\bar{x}$ is a sufficient statistic for μ by the factorization theorem

When both parameters are unknown $\begin{bmatrix}\bar{X}\\ S^2\end{bmatrix}$ is a sufficient statistic for (μ, σ^2).

Minimally Sufficient Statistic

$\forall$ 2 sample points $\underline{x}$, $\underline{y}$

$$\frac{f(\underline{x}|\theta)}{f(\underline{y}|\theta)} = \frac{\left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{-[n(\bar{x}-\mu)^2+(n-1)s_x^2]/2\sigma^2}}{\left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{-[n(\bar{y}-\mu)^2+(n-1)s_y^2]/2\sigma^2}} = e^{\frac{-n(\bar{x}^2-\bar{y}^2)+2n\mu(\bar{x}-\bar{y})-(n-1)(s_x^2-s_y^2)}{2\sigma^2}}$$

*This will be a constant function of μ, σ^2 iff $\bar{x}=\bar{y}$ & $S_x^2=S_y^2$

$\therefore$ $\begin{bmatrix}\bar{X}\\ S_x^2\end{bmatrix}$ is a minimal sufficient statistic for (μ, σ^2)

Ancillary Statistic

$X \sim N(\mu, \sigma^2)$ $\quad f_x(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \left(\frac{1}{\sigma^2 2\pi}\right)^{1/2} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

$Y \sim N(0,1)$ $\quad f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$

*We note $f_x(x)$ is a location-scale family of $f_y(y)$

ie $f_x(x) = \frac{1}{\sigma} f_y\left(\frac{x-\mu}{\sigma}\right)$

Thus $\frac{X_{(n)} - X_{(1)}}{S}$ is ancillary as the location variables cancel in both the numerator and denominator then the scale variables cancel give us a statistic related to $N(0,1)$

Complete Statistic

$$f(x|\sigma,\mu) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$= \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2\sigma^2}[x^2 - 2x\mu + \mu^2]}$$

$$= \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma^2} + \frac{x\mu}{\sigma^2} - \frac{\mu^2}{2\sigma^2}}$$

$$= \underbrace{1}_{h(x)} \underbrace{\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2\sigma^2}\mu^2}}_{c(\mu,\sigma^2)} e^{-\frac{x^2}{2\sigma^2} + \frac{x\mu}{\sigma}}$$

$t_1(x) = \sum x_i^2$, $\quad t_2(x) = \sum x_i$

$w(\mu,\sigma^2) = \frac{-1}{2\sigma^2}$ $\quad w(\mu,\sigma^2) = \frac{\mu}{\sigma}$

Thus $T = (\sum x_i^2, \sum x_i)$ is a comple statistic

Method of moments

$m_1 = \frac{1}{n}\sum_1^n X_i \qquad \mu_1 = E(x) = \mu$

$m_2 = \frac{1}{n}\sum_1^n x_i^2 \qquad \mu_2 = E(x^2) = \sigma^2 + \mu^2 \qquad (\text{as } var(x) = E(x^2) - [E(x)]^2$

Solve: $\frac{1}{n}\sum_1^n x_i = \mu \longrightarrow \hat{\mu} = \overline{X}$

$\frac{1}{n}\sum_1^n x_i^2 = \mu^2 + \sigma^2 \longrightarrow \hat{\sigma}^2 = \frac{1}{n}\sum x_i^2 - \overline{X}^2$

Maximum Likelihood Estimator

$L(\theta|\underline{x}) = f(\underline{x}|\theta) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{\frac{-1}{2\sigma^2}\sum(x_i-\mu)^2}$

$\ln(L(\theta|\underline{x})) = -n\ln(\sigma\sqrt{2\pi}) - \frac{1}{2\sigma^2}\sum(x_i-\mu)^2$

$\frac{d}{d\mu}\ln(L(\theta|\underline{x})) = \frac{d}{d\mu} - \frac{1}{2\sigma^2}\left(\sum x_i^2 - \sum 2x_i\mu + \sum\mu^2\right) = \frac{d}{d\mu}\frac{1}{\sigma^2}\sum_1^n x_i\mu - \frac{1}{2\sigma^2}\sum_1^n \mu^2$

$= \frac{1}{\sigma^2}\sum_1^n x_i - \frac{1}{\sigma^2}\sum_1^n \mu \overset{set}{=} 0$

$\frac{1}{\sigma^2}\left[\sum_1^n x_i - n\mu\right] = 0$

$\hat{\mu} = \overline{X}$

$\frac{d^2}{d\mu^2}\ln(L(\theta|x)) = -\frac{n}{\sigma^2} < 0 \quad \therefore \hat{\mu}$ is a maximum

$\frac{d}{d\sigma^2}\ln(L(\theta|x)) = \frac{d}{d\sigma^2} - n\ln(\sigma\sqrt{2\pi}) - \frac{d}{d\sigma^2}\frac{1}{2\sigma^2}\sum(x_i-\mu)^2 = \frac{d}{d\sigma^2}\frac{-n}{2}\ln(\sigma^2) - \frac{d}{d\sigma^2}\frac{n}{2}\ln(2\pi) - \frac{d}{d\sigma}\frac{1}{2\sigma^2}\sum(x_i-\mu)^2$

$= \frac{-n}{2\sigma^2} + \frac{1}{2\sigma^4}\sum(x_i-\mu)^2 \overset{set}{=} 0$

$\frac{n}{2\sigma^2} = \frac{1}{2\sigma^4}\sum(x_i-\mu)^2 \rightarrow \frac{n}{2}\sigma^2 = \frac{1}{2}\sum(x_i-\mu)^2 \rightarrow \hat{\sigma}^2 = \frac{1}{n}\sum(x_i-\mu)^2$

$\frac{d^2}{d\sigma^{2\,2}}\ln(L(\theta|x)) = \frac{n}{2\sigma^4} - \frac{1}{\sigma^6}\sum(x_i-\mu)^2\Big|_{\hat{\sigma}^2 = \frac{1}{n}\sum(x_i-\mu)^2}$

$= \frac{n^3}{2(\sum(x_i-\mu)^4)} - \frac{2n^3}{2(\sum(x_i-\mu)^4)} < 0 \quad \therefore \hat{\sigma}^2$ is a maximum

<u>Cramer Rao Lower Bound</u> For any unbiased estimator $W(x)$ $\quad var(W(x)) \geq \dfrac{\left(\frac{d}{d\theta} E_\theta(W(\underline{x}))\right)^2}{E\left(\left(\frac{d}{d\theta}\log(f(\underline{x}|\theta))\right)^2\right)}$

for σ^2

$$\frac{\left(\frac{d}{d\theta} E(W(\underline{x}))\right)^2}{E\left(\left(\frac{d}{d\theta}\log(f(\underline{x}|\theta))\right)^2\right)} = \frac{\left(\frac{d}{d\sigma^2}\sigma^2\right)^2}{E\left(\left(\frac{d}{d\theta}\log(f(\underline{x}|\theta))\right)^2\right)} = \frac{1}{E\left(\left(\frac{d}{d\theta}\log(f(\underline{x}|\theta))\right)^2\right)}$$

as $f_x(x)$ is an exp family & iid

$$= \frac{1}{-nE\left(\frac{\partial^2}{\partial\sigma^{2\,2}}\log(f(x|\theta))\right)} = \frac{1}{-nE\left(\frac{\partial^2}{\partial\sigma^{2\,2}}\log\left(\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/2\sigma^2}\right)\right)}$$

$$= \frac{1}{-nE\left(\frac{\partial^2}{\partial\sigma^{2\,2}} - \ln(\sqrt{2\pi\sigma^2}) - \frac{(x-\mu)^2}{2\sigma^2}\right)} = \frac{1}{-nE\left(\frac{\partial}{\partial\sigma^2}\frac{-1}{\sqrt{2\pi\sigma^2}}\left(\frac{\pi}{\sqrt{2\pi\sigma^2}}\right) + \frac{(x-\mu)^2}{2\sigma^4}\right)}$$

$$= \frac{1}{-nE\left(\frac{\partial}{\partial\sigma^2}\frac{-1}{2\sigma^2} + \frac{(x-\mu)^2}{2\sigma^4}\right)} = \frac{1}{-nE\left(\frac{1}{2\sigma^4} - \frac{(x-\mu)^2}{\sigma^6}\right)}$$

$$= \frac{1}{-n\left[\frac{1}{2\sigma^4} - \frac{E((x-\mu)^2)}{\sigma^6}\right]} = \frac{1}{-n\left[\frac{1}{2\sigma^4} - \frac{\sigma^2}{\sigma^6}\right]}$$

$$= \frac{1}{-n\left[\frac{1}{2\sigma^4} - \frac{1}{\sigma^4}\right]} = \frac{1}{-n\left[\frac{-1}{2\sigma^4}\right]} = \frac{2\sigma^4}{n}$$

Thus any unbiased estimator $W(\underline{x})$ of σ^2 has variance $\geq 2\sigma^4/n$

for μ

$$\frac{\left(\frac{d}{d\theta} E(W(\underline{x}))\right)^2}{E\left(\left(\frac{d}{d\theta}\log(f(\underline{x}))\right)^2\right)} = \frac{\left(\frac{d}{d\mu}\mu\right)^2}{E\left(\left(\frac{d}{d\theta}\log(f(\underline{x}))\right)^2\right)} = \frac{1}{E\left(\left(\frac{d}{d\theta}\log(f(\underline{x}))\right)^2\right)}$$

as $f_x(x)$ is an exp. family & iid

$$= \frac{1}{-nE\left(\frac{\partial^2}{\partial\mu^2}\log(f(x))\right)} = \frac{1}{nE\left(\frac{\partial^2}{\partial\mu^2}\log\left(\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(x-\mu)^2}{2\sigma^2}}\right)\right)} = \frac{1}{-nE\left(\frac{\partial^2}{\partial\mu^2} - \ln(\sigma\sqrt{2\pi}) - \frac{(x-\mu)^2}{2\sigma^2}\right)}$$

$$= \frac{1}{nE\left(\frac{\partial^2}{\partial\mu^2}\frac{(x-\mu)^2}{2\sigma^2}\right)} = \frac{1}{nE\left(\frac{\partial}{\partial\mu}\left(\frac{x-\mu}{\sigma^2}\right)\right)} = \frac{1}{nE\left(\frac{1}{\sigma^2}\right)} = \frac{1}{n\left(\frac{1}{\sigma^2}\right)} = \frac{\sigma^2}{n}$$

Thus any unbiased estimator $W(\underline{x})$ of σ^2 has variance $\geq \frac{\sigma^2}{n}$

Likelihood Ratio Test

$H_0: \theta \in \Theta_0$ w/ iid sample $x_1, x_2, \ldots x_n$

$H_a: \theta \in \Theta_0^c$

$$\lambda(\underline{x}) = \frac{\sup_{\Theta_0} L(\theta|\underline{x})}{\sup_{\Theta} L(\theta|\underline{x})} = \frac{L(\breve{\theta}|\underline{x})}{L(\hat{\theta}|\underline{x})}$$

$$L(\theta|\underline{x}) = \prod \frac{1}{\sigma\sqrt{2\pi}} e^{-(x_i-\mu)^2/2\sigma^2} = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{-\frac{1}{2\sigma^2}\sum_1^n (x_i-\mu)^2}$$

so, $$\lambda(\underline{x}) = \frac{\left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{-1/2\sigma^2 \sum_1^n (x_i-\breve{\theta})^2}}{\left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{-1/2\sigma^2 \sum_1^n (x_i-\hat{\theta})^2}} = e^{-\frac{1}{2\sigma^2}\left[\sum_1^n (x_i-\breve{\theta})^2 - \sum_1^n (x_i-\hat{\theta})^2\right]}$$

$$= e^{\frac{-1}{2\sigma^2}\left[\sum_1^n x_i^2 - 2\sum_1^n \breve{\theta} x_i + \sum_1^n \breve{\theta}^2 - \sum_1^n x_i^2 + 2\sum_1^n \hat{\theta} x_i - \sum_1^n \hat{\theta}^2\right]}$$

$$= e^{\left[(-2\breve{\theta} + 2\hat{\theta})\sum_1^n x_i + n(\breve{\theta}^2 - \hat{\theta}^2)\right]}$$ <u>for μ</u>

so, $$\lambda(\underline{x}) = \frac{\frac{1}{\sqrt{2\pi\breve{\theta}}} e^{\frac{-1}{2\breve{\theta}}\sum_1^n (x_i-\mu)^2}}{\frac{1}{\sqrt{2\pi\hat{\theta}}} e^{\frac{-1}{2\hat{\theta}}\sum_1^n (x_i-\mu)^2}} = \frac{\frac{1}{\sqrt{\breve{\theta}}}}{\frac{1}{\sqrt{\hat{\theta}}}} e^{\frac{-1}{2\breve{\theta}}\sum_1^n (x_i-\mu)^2 + \frac{1}{2\hat{\theta}}\sum_1^n (x_i-\mu)^2}$$

$$= \sqrt{\frac{\hat{\theta}}{\breve{\theta}}}\, e^{\left(\sum_1^n (x_i-\mu)^2\right)\left(\frac{1}{2\hat{\theta}} - \frac{1}{2\breve{\theta}}\right)}$$ <u>for σ^2</u>