Chapter 27 Repeated Measures

- Repeated Measures utilize the same expiramental unit for each treatment under the study
- Provides good precision - experimental units count as their own control essentially
- Need to watch out for order and carryover effect

Single-Factor Repeated Measures

Treatment Order

	1	2	3	4
subject 1	T_4	T_3	T_2	T_1
2	T_3	T_4	T_1	T_2
3	T_4	T_3	T_1	T_2
4	T_2	T_1	T_4	T_3
5	T_1	T_2	T_4	T_3

$Y_{ij} = \mu_{..} + \rho_i + \tau_j + \epsilon_{ij}$

where: $\mu_{..}$ is a constant

ρ_i are ind. $N(0, \sigma_\rho^2)$

τ_j are constants $\Sigma \tau_j = 0$

ϵ_{ij} are ind. $N(0, \sigma^2)$

$\rho_i \perp \epsilon_{ij}$

$E(Y_{ij}) = \mu_{..} + \tau_j$

$\sigma^2(Y_{ij}) = \sigma_Y^2 = \sigma_\rho^2 + \sigma^2$

$\sigma(Y_{ij}, Y_{ij'}) = \sigma_\rho^2 = \omega \sigma_Y^2 \rightarrow \omega = \frac{\sigma_\rho^2}{\sigma_Y^2}$

$\sigma(Y_{ij}, Y_{i'j'}) = 0$

Analysis of Variance

SSTO = SSSubjects + SSTreatments + SSTreatment*Sub

$SSTO = \sum_i \sum_j (Y_{ij} - \bar{Y}_{..})^2$

$SSSubjects = r \sum_i (\bar{Y}_{i.} - \bar{Y}_{..})^2$

$SSTreatments = s \sum_j (\bar{Y}_{.j} - \bar{Y}_{..})^2$

$SSTreatments^*sub = \sum_i \sum_j (Y_{ij} - \bar{Y}_{i.} - \bar{Y}_{.j} + \bar{Y}_{..})^2$

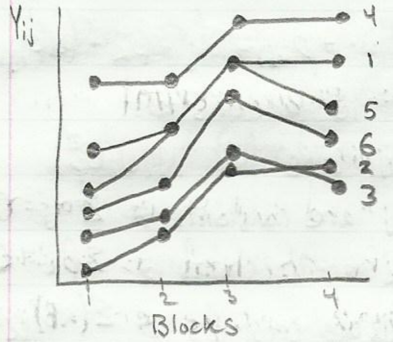
Test for Treatment Effects

H_0: all $\tau_j = 0$

H_a: not H_0

$F^* = MSTreatments / MSTreatment^*sub$

Reject when $> F[1-\alpha, r-1, (r-1)(s-1)]$



Note: • 1 is always the lowest
• 3 or 4 is always highest
• No gross deviation from parallel lines

Diagnostics are the same as RCBD 1) Check Y_{ij} vs. Subjects
2) Check Y_{ij} vs. $\hat{Y}_{ij}$
3) Normal Probability Plot

$$e_{ij} = Y_{ij} - \bar{Y}_{i\cdot} - \bar{Y}_{\cdot j} + \bar{Y}_{\cdot\cdot}$$

Analysis of Treatment effects $s^2(\hat{L}) = MSTreatment^{*}sub\left(\frac{1}{s} + \frac{1}{s}\right)$ s = # of subjects

Confidence Interval: $\hat{L} \pm \frac{1}{2} q(1-\alpha, r-1, (r-1)(s-1))$ Tukey

Ranked Data H_0: all $\tau_j = 0$

H_a: not H_0

$F^* = MSTreatments / MSBlock{*}Treatment$

Reject if $F^* > F(1-\alpha, r-1, (r-1), (r-1)(s-1))$

Rank F or Friedman Test $\chi^2_F = SSTreatment \div \frac{SSTreatment + SSBlock{*}treatment}{n_b(r-1)}$

Look at: $\underbrace{\bar{R}_{\cdot j} - \bar{R}_{\cdot j'} \pm B\left[\frac{r(r+1)}{6s}\right]^{1/2}}$ w/ $B = z(1-\alpha/2g)$

$g = \frac{r(r-1)}{2}$

If this contains 0 we assume $\mu_{\cdot j}$ and $\mu_{\cdot j'}$ do not differ

Two-Factor Repeated Measures: $Y_{ijk} = \mu_{\cdots} + \rho_{i(j)} + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \epsilon_{ijk}$

A	subject	Treatment order 1	2
A_1	1	A_1B_1	A_1B_2
	⋮		⋮
	s	A_1B_1	A_1B_2
A_2	s+1	A_2B_2	A_2B_1
	⋮		⋮
	2s	A_2B_1	A_2B_2

- $\mu_{\cdots}$ is a constant
- $\rho_{i(j)}$ are ind $N(0, \sigma_\rho^2)$
- α_j are constants $\ni \sum \alpha_j = 0$
- β_k are constants $\ni \sum \beta_k = 0$
- $(\alpha\beta)_{jk}$ are constants $\ni \sum(\alpha\beta)_{jk} = 0$
- $\epsilon_{ijk} \sim N(0, \sigma^2)$
- $\rho_{i(j)} \perp\!\!\!\perp \epsilon_{ijk}$
- NO INTERACTIONS!

$$E(Y_{ijk}) = \mu_{\cdots} + \alpha_j + \beta_k + (\alpha\beta)_{jk}$$
$$\sigma^2(Y_{ijk}) = \sigma_Y^2 = \sigma_\rho^2 + \sigma^2$$
$$\sigma(Y_{ijk}, Y_{ijk'}) = \sigma_\rho^2$$
$$\sigma(Y_{ijk}, Y_{i'j'k'}) = 0$$

Sum of Squares - see table 27.5 p. 1142
Mean Squares - see table 27.6 p. 1143

Tests for factor Effects
1. H_0: all $(\alpha\beta)_{jk} = 0$
   H_a: not H_0
   $F^* = MSAB/MSE$
   Reject if $> F[1-\alpha, (a-1)(b-1), a(s-1)(b-1)]$
2. H_0: all $\alpha_j = 0$
   H_a: not H_0
   $F^* = MSA/MS\text{ Subjects in factor A}$
   Reject if $F^* > F[1-\alpha, a-1, a(s-1)]$
3. H_0: all $\beta_k = 0$
   H_a: not H_0
   $F^* = MSB/MSE$
   Reject if $F^* > F[1-\alpha, b-1, a(s-1)(b-1)]$

Diagnostics · $e_{ijk} = Y_{ijk} - \bar{Y}_{\cdot jk} - \bar{Y}_{ij\cdot} + \bar{Y}_{\cdot j\cdot}$

· Check same plots as single factor model

Analysis of Factor Effects

$$\left.\begin{aligned} L &= \mu_{\cdot\cdot k} - \mu_{\cdot\cdot k'} \\ \hat{L} &= Y_{\cdot\cdot k} - Y_{\cdot\cdot k'} \end{aligned}\right\} \text{Factor B}$$

Confidence Intervals:

$\hat{L} \pm t[1-\alpha/2,\ a(s-1)(b-1)]s(\hat{L})$ single comp.

$\hat{L} \pm \frac{1}{\sqrt{2}} q[1-\alpha,\ b,\ a(s-1)(b-1)]s(\hat{L})$ Pairwise - Tukey

$\hat{L} \pm (b-1)F[1-\alpha,\ b-1,\ a(s-1)(b-1)]s(\hat{L})$ Scheffe

$\hat{L} \pm t[1-\alpha/2g,\ a(s-1)(b-1)]s(\hat{L})$ Bonferroni

$$\left.\begin{aligned} L &= \mu_{\cdot j\cdot} - \mu_{\cdot j'\cdot} \\ \hat{L} &= \bar{Y}_{\cdot j\cdot} - \bar{Y}_{\cdot j'\cdot} \end{aligned}\right\} \text{Factor A}$$

Confidence Intervals:

$\hat{L} \pm t[1-\alpha/2,\ a(s-1)]s(\hat{L})$ single comp

$\hat{L} \pm \frac{1}{\sqrt{2}} q[1-\alpha,\ a,\ a(s-1)]s(\hat{L})$ Pairwise Tukey

$\hat{L} \pm (a-1)F[1-\alpha,\ a-1,\ a(s-1)]s(\hat{L})$ Scheffe

$\hat{L} \pm t[1-\alpha/2g,\ a(s-1)]s(\hat{L})$ Bonferroni

With Interactions

- Analysis becomes very complex
- Test H_0: all $(\alpha\beta)_{jk} = 0$ as before
- With interactions we fail to reject H_0
- Any means that vary by more than $\frac{1}{\sqrt{2}} q(1-\alpha,\ \text{levels of A},\ df_{adj})$ $df_{adj} \rightarrow$ page 1152

Blocking in Repeated Measures

		Treatment order 1	2
Block 1	S_1 S_2		
Block 2	S_3 S_4		
⋮	⋮		
Block n_b	S_{2n_b-1} S_{2n_b}		

· Used to improve precision

Two-Factor Repeated Measures on both factors

$Y_{ijk} = \mu_{\cdots} + \rho_i + \alpha_j + \beta_k + (\alpha\beta)_{jk} + (\rho\alpha)_{ij} + (\rho\beta)_{ik} + \epsilon_{ijk}$

- $\mu_{\cdots}$ is constant
- ρ_i are ind $N(0, \sigma_\rho^2)$
- α_j are constants $\ni \Sigma\alpha_j = 0$
- β_k are constants $\ni \Sigma\beta_k = 0$
- $(\alpha\beta)_{jk}$ are constants $\ni \Sigma(\alpha\beta)_{jk} = 0$
- $(\rho\beta)_{ik} \sim N(0, \frac{b-1}{b}\sigma^2_{\rho\beta}) \ni \Sigma(\rho\beta)_{ik} = 0$
- $\sigma((\rho\beta)_{ik}, (\rho\beta)_{ik'}) = -\frac{1}{b}\sigma^2_{\rho\beta}$
- $(\rho\alpha)_{ij} \sim N(0, \frac{a-1}{a}\sigma^2_{\rho\alpha}) \ni \Sigma(\rho\alpha) = 0$
- $\sigma((\rho\alpha)_{ij}, (\rho\alpha)_{ij'}) = -\frac{1}{a}\sigma^2_{\rho\alpha}$
- $\rho_i, (\rho\alpha)_{ij}, (\rho\beta)_{ik}$ are pairwise $\perp\!\!\!\perp$

Treatment Order

	1	2	3	4
Sub. 1	A_1B_2	A_2B_2	A_1B_1	A_2B_1
2	A_2B_1	A_1B_2	A_2B_2	A_1B_1
3	A_2B_2	A_1B_1	A_2B_1	A_1B_2
4	A_1B_1	A_2B_1	A_1B_2	A_2B_2

$$E(Y_{ijk}) = \mu_{\cdots} + \alpha_j + \beta_k + (\alpha\beta)_{jk}$$

$$\sigma^2(Y_{ijk}) = \sigma_Y^2 = \sigma_\rho^2 + \left(\frac{a-1}{a}\right)\sigma^2_{\rho\alpha} + \left(\frac{b-1}{b}\right)\sigma^2_{\rho\beta} + \sigma^2$$

Sum of squares table 27.11 – p. 1156

Test for Factor Effects

① H_0: all $(\alpha\beta)_{jk} = 0$

H_a: not H_0

$F^* = MSAB/MSE$

Reject if $F^* > F[1-\alpha, (a-1)(b-1), (a-1)(b-1)(s-1)]$

② H_0: all $\beta_k = 0$

H_a: not H_0

$F^* = MSB/MSBS$

Same Diagnostics

Analysis of Factor Effects

$$\hat{L} = \mu_{.jk} - \mu_{.j'k'}$$

$$s^2(\hat{L}) = MSABS\left(\frac{1}{s} + \frac{1}{s}\right)$$

confidence Intervals for Factor B

$\hat{L} \pm t[1-\alpha/2, (b-1)(s-1)]\, s(\hat{L})$ single comp

$\hat{L} \pm \frac{1}{\sqrt{2}} q[1-\alpha, b, (b-1)(s-1)]\, s(\hat{L})$ pairwise - Tukey

$\hat{L} \pm (b-1) F[1-\alpha, b-1, (b-1)(s-1)]\, s(\hat{L})$ scheffe

$\hat{L} \pm t[1-\alpha/2g, (b-1)(s-1)]\, s(\hat{L})$ Bonferroni

confidence Intervals for Factor A

$\hat{L} \pm t[1-\alpha/2, (a-1)(s-1)]\, s(\hat{L})$ single comp

$\hat{L} \pm \frac{1}{\sqrt{2}} q[1-\alpha, a, (a-1)(s-1)]\, s(\hat{L})$ pairwise - tukey

$\hat{L} \pm (a-1) F[1-\alpha, a-1, (a-1)(s-1)]\, s(\hat{L})$ scheffe

$\hat{L} \pm t[1-\alpha/2g, (a-1)(s-1)]\, s(\hat{L})$ Bonferroni

Split Plot Design -

Factor A (Whole Plots)

factor B Split Plots

A_2B_2	A_1B_1	A_1B_2	A_2B_1
A_2B_1	A_1B_2	A_1B_1	A_2B_2
A_2	A_1	A_1	A_2

SAS See code from Chapter 25

```
/* You can code split plots w/ or without nesting,
   this is without nesting. With nesting is provided
   in sas files*/
proc covtest;
   class  A  B C;
   model  y = A  C  B*C;  */Factorial Design*/
   random A;
run;
```