$\boxed{\text{Chapter 23}}$ Unequal sample sizes

Number of cases for i^{th} factor level of A $= n_{i\cdot} = \sum_j n_{ij}$

Number of cases for j^{th} factor level of B $= n_{\cdot j} = \sum_i n_{ij}$

Total number of cases $= n_T = \sum_i \sum_j n_{ij}$

$\bar{Y}_{ij\cdot} = \frac{Y_{ij\cdot}}{n_{ij}}$ where $Y_{ij\cdot} = \sum_{k=1}^{n_{ij}} Y_{ijk}$

Two-Factor ANOVA

$$\left.\begin{aligned} Y_{ijk} &= \mu_{\cdot\cdot} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ij} \\ E(Y_{ijk}) &= \mu_{\cdot\cdot} + \alpha_i + \beta_j + (\alpha\beta)_{ij} \end{aligned}\right\} \text{from chapter 19}$$

*Now, $Y_{ijk} = \mu_{\cdot\cdot} + \alpha_1 X_{ijk1} + \beta_1 X_{ijk2} + \beta_2 X_{ijk3} + (\alpha\beta)_{11} X_{ijk1}X_{ijk2} + (\alpha\beta)_{12} X_{ijk1}X_{ijk2} + \epsilon_{ijk}$

where $X_1 = \begin{matrix} 1 & \text{level 1 factor A} \\ -1 & \text{level 2 factor A} \end{matrix}$, $X_2 = \begin{matrix} 1 & \text{level 1 B} \\ -1 & \text{level 3 B} \\ 0 & \text{ow} \end{matrix}$, $X_3 = \begin{matrix} 1 & \text{level 2 B} \\ -1 & \text{level 3 B} \\ 0 & \text{ow} \end{matrix}$

$\mu_{\cdot\cdot}$, $\alpha = \mu_{1\cdot} - \mu_{\cdot\cdot}$, $\beta_1 = \mu_{\cdot 1} - \mu_{\cdot\cdot}$, $\beta_2 = \mu_{\cdot 2} - \mu_{\cdot\cdot}$, $(\alpha\beta)_{11} = \mu_{11} - \mu_{1\cdot} - \mu_{\cdot 1} + \mu_{\cdot\cdot}$,

$(\alpha\beta)_{12} = \mu_{12} - \mu_{1\cdot} - \mu_{\cdot 2} + \mu_{\cdot\cdot}$

Hyp. Test ① H_0: all $(\alpha\beta)_{ij} = 0$ $\qquad Y_{ijk} = \mu_{\cdot\cdot} + \alpha_1 X_{ijk1} + \beta_1 X_{ijk2} + \beta_2 X_{ijk3} + \epsilon_{ijk}$ (Reduced)

H_a: not H_0

$$F^* = \frac{SSE(R) - SSE(F)}{df_R - df_F} \div \frac{SSE(F)}{df_F}$$

Reject for $F^* > F(1-\alpha, df_R - df_F, df_F)$

Hyp Test ② H_0: all $\alpha_i = 0$ | ③ H_0: all $\beta_j = 0$

H_a: not H_0 | H_a: not H_0

$Y_{ij} = \mu_{\cdot\cdot} + \beta_1 X_{ijk2} + \beta_2 X_{ijk3} + (\alpha\beta)_{11} X_{ijk1} X_{ijk2} + (\alpha\beta)_{12} X_{ijk1} X_{ijk3} + \epsilon_{ijk}$ (Reduced) | $Y_{ij} = \mu_{\cdot\cdot} + \alpha_1 X_{ijk1} + (\alpha\beta)_{11} X_{ijk1} X_{ijk2} + (\alpha\beta)_{12} X_{ijk1} X_{ijk3} + \epsilon_{ijk}$ (Reduced)

$$F^* = \frac{SSE(R) - SSE(F)}{df_R - df_F} \div \frac{SSE(F)}{df_F}$$

Reject for $F^* > F(1-\alpha, df_R - df_F, df_F)$

① H_0 true $\rightarrow \frac{MSE(AB|A,B)}{MSE(A,B,AB)} \sim F((a-1)(b-1), df_E)$

② H_0 true $\rightarrow \frac{MSE(A|B,AB)}{MSE(A,B,AB)} \sim F(a-1, df_E)$

③ H_0 true $\rightarrow \frac{MSE(B|A,AB)}{MSE(A,B,AB)} \sim F(b-1, df_E)$

* Page 961 — Table 23.5 has the formulas for all point estimators, variance estimators and confidence interval multipliers
  - We covered this on pg 88 for RCBD but here we'll just defer to SAS

SAS
```
proc glm plots=all;          /* check if we Type III can drop interactions */
    class x1 x2;             /* then we check other factors */
    model y= x1|x2;
run;
proc glm plots=all;
    class x1 x2;
    model y= x1 x2;
    lsmeans x1 / pdiff adjust=tukey α=.05 cl;   /* all pairwise diff. */
    lsmeans x2 / pdiff adjust=tukey α=.05 cl;
run;
proc glimmix plots=all; /* linear combinations */
    class x1 x2;
    model y= x1 x2;
    lsmestimate x1 "M1-M2" 1 -1 0 /adjust=scheffe/bonf α=.05 cl;
run
```

Skipped 23.4 - 23.6