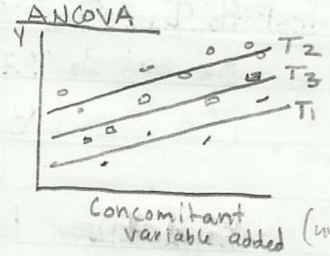
Chapter 22 ANCOVA



* Reduced Variability

(unaffected by treatments) pre study

n_i = number of cases in the i^{th} level

$n_T = \Sigma n_i$

$Y_{ij} = \mu_{\cdot} + \tau_i + \epsilon_{ij}$ ANOVA

In text, we fit it differently

$$\underline{Y_{ij}} = \mu_{\cdot} + \tau_i + \gamma(X_{ij} - \bar{X}_{\cdot\cdot}) + \epsilon_{ij} = \mu_{\cdot} + \tau_1 I_{ij1} + \tau_2 I_{ij2} \cdots + \gamma x_{ij} + \epsilon_{ij} \quad \text{ANCOVA}$$

$I_{ij1} = \begin{cases} 1 & \text{treat 1} \\ -1 & \text{treat 3} \\ 0 & \text{ow} \end{cases}$ $\quad I_{ij2} = \begin{cases} 1 & \text{treat 2} \\ -1 & \text{treat 3} \\ 0 & \text{ow} \end{cases}$

$\mu_{\cdot}$ = overall mean

τ_i = fixed treatment effects $\rightarrow \Sigma \tau_i = 0$

γ = regression coefficient for the relation between Y and X

X_{ij} are constants (concomitant observations)

$\epsilon_{ij} \overset{ind}{\sim} N(0, \sigma^2)$

$i = 1, \ldots, r$; $j = 1, \ldots, n_i$

$E(Y_{ij}) = \mu_{\cdot} + \tau_i + \gamma(X_{ij} - \bar{X}_{\cdot\cdot})$

$\sigma^2(Y_{ij}) = \sigma^2$

[Note: this depends on concomitant variables so all observations of treatment won't have the same mean response]

- We can generalize this model for X's that are random variables
- We can generalize for nonlinear Y, X relationships
  - ie quadratic: $Y_{ij} = \mu_{\cdot} + \tau_i + \gamma_1(X_{ij1} - \bar{X}_{\cdot\cdot 1}) + \gamma_2(X_{ij2} - \bar{X}_{\cdot\cdot 2}) + \epsilon_{ij}$
- We can add more concomitant observations
  - ie two: $Y_{ij} = \mu_{\cdot} + \tau_i + \gamma_1(X_{ij1} - \bar{X}_{\cdot\cdot 1}) + \gamma_2(X_{ij2} - \bar{X}_{\cdot\cdot 2}) + \epsilon_{ij}$

Assumptions
1. Normality of error terms
2. Constant error variance across treatments
3. Equality of slope of different treatment regression lines
4. Linearity of regression lines
5. Uncorrelated error terms

Hyp Test: $H_0: \tau_1 = \tau_2 = \cdots \tau_r = 0$

H_a: not H_0

$$F^* = \frac{MSR(X_q, \ldots X_{p-1} \mid X_1, \ldots, X_{q-1})}{MSE}$$

Reject for large F^*

* If we fail to reject we're interested in looking at linear combinations of regression coefficients to investigate these effects.
* Look at Type III tests of fixed effects is SAS below

SAS

```
proc sgscatter; /* Gives Ancova scatterplot linear -> no interaction */
  plot y * concomitant / group= x loess;
run;
proc glimmix; /* Get regression equation */
  class x;
  model y = x concomitant / solution; /* Yij = μ.. + τ1 Iij1 + ... + γ xij + εij */
  lsmestimate x "μ1 - μ2" 1 -1 0 / adjust = t cl alpha = .05; /* treat diff */
  lsmeans x; / gives μ1, μ2, ... estimates
run;
proc glimmix; /* test x* concomitant = 0 ≡ parallel lines in Ancova scatter */
  class x;
  model y = x | concomitant;
run;
```

Covariance Model for Two Factors

$Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} - \epsilon_{ijk}$ ANOVA

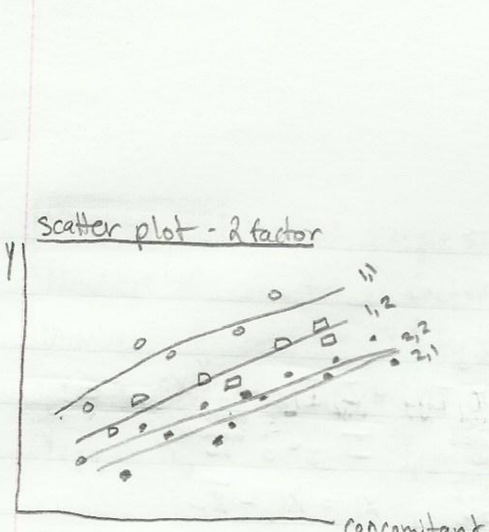
$Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \gamma(X_{ijk} - \bar{X}_{...}) + \epsilon_{ijk}$

Intext, we fit it differently $\rightarrow = \mu_{..} + \alpha_1 I_{ijk1} + \beta_1 I_{ijk2} + (\alpha\beta)_{11} I_{ijk1} I_{ijk2} + \gamma X_{ijk} + \epsilon_{ijk}$

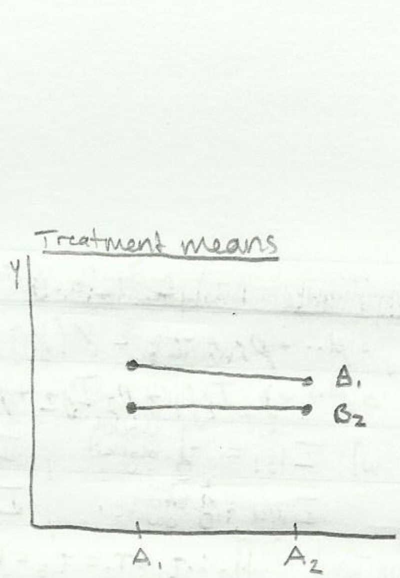
$I_1 = \begin{cases} 1 & \text{level 1 factor A} \\ -1 & \text{level 2 factor A} \end{cases}$

$I_2 = \begin{cases} 1 & \text{level 1 factor B} \\ -1 & \text{level 2 factor B} \end{cases}$

scatter plot - 2 factor



Treatment means



Hyp Test

H_0: no an interaction

H_a: $\exists$ interaction

$$F^* = \frac{SSR(I_1 I_2 \mid X, I_1, I_2)}{MSE}$$

Reject for large F^*

Linear Combinations $\left.\begin{matrix}\alpha_1 - \alpha_2 \\ \beta_1 - \beta_2\end{matrix}\right\}$ Comparing main effects factor levels

$\boxed{\text{SAS}}$

```
data twoFactor;
    set ____;
    group='11';
    if x1=1 and x2=2 then group='12';
    if x1=2 and x1=1 then group='21';
    if x1=2 and x2=2 then group='22';
run;
proc sgscatter data=twoFactor;
    plot y*concomitant /group=group loess;
run;
proc glm; /*check type 3 test for iteraction*/
    class x1 x2;
    model y=concomitant x1|x2 /solution;
run;
proc glimmix; class x1 x2; /*Comparing main effects*/
    model y=concomitant x1 x2 /solution;
    lsmestimate x1 "α1-α2" 1 -1 /cl α=.05;
    lsmestimate x2 "β1-β2" 1 -1 /cl α=.05; run;
```

Covariance Analysis of RCBD

$$Y_{ij} = \mu_{\cdot\cdot} + \rho_i + \tau_j + \gamma(X_{ij} - \overline{X}_{\cdot\cdot}) + \epsilon_{ij}$$
$$= \mu_{\cdot\cdot} + \rho_1 I_{ij1} + \rho_2 I_{ij2} + \rho_3 I_{ij3} + \tau_1 I_{ij4} + \tau_2 I_{ij5} + \gamma x_{ij} + \epsilon_{ij}$$

w/ $I_{ij1} = \begin{cases} 1 & \text{block1} \\ -1 & \text{block4} \\ 0 & \text{ow} \end{cases}$, $I_{ij2} = \begin{cases} 1 & \text{block2} \\ -1 & \text{block3} \\ 0 & \text{ow} \end{cases}$, $I_{ij3} = \begin{cases} 1 & \text{block3} \\ -1 & \text{block4} \\ 0 & \text{ow} \end{cases}$,

$I_{ij4} = \begin{cases} 1 & \text{Treat1} \\ -1 & \text{Treat3} \\ 0 & \text{ow} \end{cases}$, $I_{ij5} = \begin{cases} 1 & \text{treat2} \\ -1 & \text{treat3} \\ 0 & \text{ow} \end{cases}$, $x_{ij} = X_{ij} - \overline{X}_{\cdot\cdot}$

Hyp Test $H_0: \tau_1 = \tau_2 = \tau_3 = 0$

H_a: not H_0.

Use p value in type III table