Chapter 21

<u>Randomized Complete Block Design - RCBD</u> - Experimental units are sorted into homogeneous groups, called blocks, and all treatment combinations are assigned at random to experimental units within the blocks

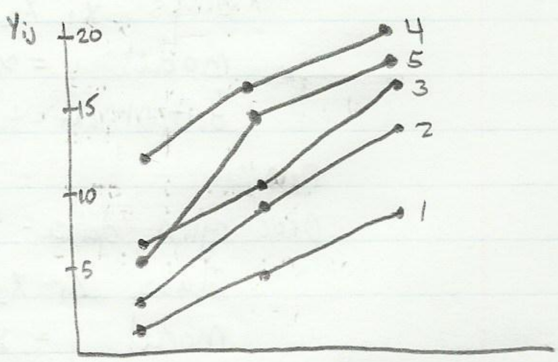
<u>Blocking Criteria</u>
- Characteristics of experimental units
- Characteristics of experimental setting

RCBD ADVANTAGES	RCBD DISADVANTAGES
· can provide more precise results	· Complex if observations are missing within a block
· can accomodate any # of treatments	· Smaller degrees of freedom for experimental error
· can have unequal samplesizes	· More assumptions: no interactions, block to block constant variance
· can drop whole blocks if necessary	
· can deliberately widen range of validity	

<u>RCBD Example</u>

		Exp. Unit 1	2	3
Block	1	C	W	U
	2	C	U	W
	3	U	W	C
	4	W	U	C
	5	W	C	U

w/ treatments W, U, C



Note:
- Variations between blocks
- No important interactions apparant (almost parallel)

RCBD Model: $Y_{ij} = \mu_{..} + \rho_i + \tau_j + \epsilon_{ij} \quad \ni \Sigma \rho_i = 0, \Sigma \tau = 0, \epsilon \sim N(0, \sigma^2)$

constant, constant for block effects, constant for treatment effects

$E(Y_{ij}) = \mu_{..} + \rho_i + \tau_j$

$\sigma^2(Y_{ij}) = \sigma^2$

Estimators
$\hat{\mu}_{..} = \bar{Y}_{..}$

$\hat{\rho}_i = \bar{Y}_{i.} - \bar{Y}_{..}$

$\hat{\tau}_j = \bar{Y}_{.j} - \bar{Y}_{..}$

$\epsilon_{ij} = Y_{ij} - \hat{Y}_{ij} = Y_{ij} - \bar{Y}_{i.} - \bar{Y}_{.j} + \bar{Y}_{..}$

ANOVA
SSBlock $= r\, \Sigma(\bar{Y}_{i.} - \bar{Y}_{..})^2$ $\quad n_b - 1$ deg. of freedom

SSTreatment $= n_b\, \Sigma(\bar{Y}_{.j} - \bar{Y}_{..})^2$ $\quad r-1$ deg. of freedom

SSE = SSBLTR $= \sum_i \sum_j (Y_{ij} - \bar{Y}_{i.} - \bar{Y}_{.j} + \bar{Y}_{..})^2 = \sum_i \sum_j e_{ij}^2$ $\quad (n_b-1)(r-1)$ deg. of freedom

MSBlock = SSBlock$/(n_b-1)$

MSTreatment = SSTreatment$/(r-1)$

MSE = MSBLTR = SSBLTR$/((n_b-1)(r-1))$

Hyp. Test
H_0: all $\tau_j = 0$ $\quad$ Look at Type III sum of squares

H_a: not H_0

1) $F^* = \frac{MSTreatment}{MSBL.TR}$

If $F^* > F[1-\alpha, r-1, (n_b-1)(r-1)]$ reject

2) χ_F^{2*} = SSTreatment $\div \frac{SSTreatment + SSBLTR}{n_b(r-1)}$ $\quad$ [Friedman test]

If $\chi_F^{2*} > \chi^2(1-\alpha, r-1)$ reject

3) $F^*_{Rank} = \frac{(n_b-1)\chi_F^2}{n_b(r-1) - \chi_F^2}$

$\boxed{SAS}$
```
proc glm plots=all data=____;
    class x1 x2;
    model y=x1 x2;
    lsmeans x1 x2;   <-- Add   output out=diag r=resid;  for diag
run;
proc sgscatter data=diag;
    resid * (x1 x2);        check constant variance in blocks & treatment
run;
```

<u>Tukey's Additivity Test</u> See page 85

$$F^* = \frac{\left(\Sigma\Sigma(\bar{Y}_{i.} - \bar{Y}_{..})(\bar{Y}_{.j} - \bar{Y}_{..})Y_{ij}\right)^2}{\left(\Sigma(\bar{Y}_{i.} - \bar{Y}_{..})^2\right)\left(\Sigma(\bar{Y}_{.j} - \bar{Y}_{..})^2\right)} \div \frac{SSTO - SSBL - SSTR - SSBLTR}{rn_b - r - n_b}$$

<u>Pairwise Comparisons</u> see page 83

$s^2(\hat{L}) = MSBLTR\left(\frac{2}{n_b}\right)$

<u>Single comparison</u>	$\pm\ s^2(\hat{L}) * t[1-\alpha/2, (n_b-1)(r-1)]$
<u>Tukey</u>	$\pm\ s^2(\hat{L}) * \frac{1}{\sqrt{2}} q[1-\alpha, r, (n_b-1)(r-1)]$
<u>Scheffe</u>	$\pm\ s^2(\hat{L}) * (r-1)F[1-\alpha, r-1, (n_b-1)(r-1)]$
<u>Bonferroni</u>	$\pm\ s^2(\hat{L}) * t[1-\alpha/2g, (n_b-1)(r-1)]$

<u>Generalized Randomized Block Design</u> $Y_{ijk} = \mu_{..} + \rho_i + \tau_j + (\rho\tau)_{ij} + \epsilon_{ijk}$

* Same as CRBD except we allow interaction $(\rho\tau)_{ij} \ni \Sigma(\rho\tau)_{ij} = 0$

<u>Factorial Treatments - Two factor RCBD</u> $Y_{ijk} = \mu_{..} + \rho_i + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \epsilon_{ijk}$

Block	A_1 B_1	A_1 B_2	A_2 B_1	A_2 B_2
1	Y_{111}	Y_{112}	Y_{121}	Y_{122}
2	Y_{211}	Y_{212}	Y_{221}	Y_{222}
3	Y_{311}	Y_{312}	Y_{321}	Y_{322}

SS	d.f
SSBL	$n_b - 1$
SSTR	$r - 1$
SSA	$a - 1$
SSB	$b - 1$
SSAB	$(a-1)(b-1)$
SSBLTR	$(n_b-1)(r-1)$
SSTO	$n_b r - 1$

SSTR = SSA + SSB + SSAB

SAS

```
Proc Anova;
   class x1 x2 x3;
   model y= x1 x2|x3;
run;
```