$\boxed{\text{Chapter 17}}$ 17.1 & 17.2 omitted

- **Inferences for factor level means:**
  1) A single factor level mean
  2) A difference between two factor level means
  3) A contrast among factor level means
  4) A linear combination of factor level means

① • An unbiased estimator of μ_i is $\hat{\mu}_i = \bar{Y}_{i\cdot}$

$E(\bar{Y}_{i\cdot}) = \mu_i$

$\sigma^2(\bar{Y}_{i\cdot}) = \sigma^2/n_i$

$S^2(\bar{Y}_{i\cdot}) = MSE/n_i$

Note: • $\dfrac{\bar{Y}_{i\cdot} - \mu_i}{s(\bar{Y}_{i\cdot})} \sim t(n_T - r)$

• $SSE/\sigma^2 \sim \chi^2_{(n_T - r)}$

• Hypothesis Test: $H_0: \mu_i = c$

$H_a: \mu_i \neq c$

$$t^* = \frac{\bar{Y}_{i\cdot} - c}{s(\bar{Y}_{i\cdot})}$$

Reject if $|t^*| > t(1-\alpha/2, n_T - r)$

② • $D = \mu_i - \mu_{i'}$ estimated by $\hat{D} = \bar{Y}_{i\cdot} - \bar{Y}_{i'\cdot}$

$E(\hat{D}) = \mu_i - \mu_{i'}$

$\sigma^2(\hat{D}) = \sigma^2 \left(\frac{1}{n_i} + \frac{1}{n_{i'}}\right)$

$S^2(\hat{D}) = MSE \left(\frac{1}{n_i} + \frac{1}{n_{i'}}\right)$

Note $\dfrac{\hat{D} - D}{s(\hat{D})} \sim t(n_T - r)$

Hypothesis Test $H_0: \mu_i = \mu_{i'} \equiv \mu_i - \mu_{i'} = 0$

$H_a: \mu_i \neq \mu_{i'} \equiv \mu_i - \mu_{i'} \neq 0$

$$t^* = \frac{\hat{D}}{s(D}$$

Reject if $|t^*| > t(1-\alpha/2, n_T - r)$

③ Contrast - a comparison involving two or more factor level means and includes the previous case of a pairwise difference between two factor level means

Denote $L = \sum_i^r c_i \mu_i$ where $\sum_i c_i = 0$

example 1: $L = \mu_1 - \mu_2$ (Note this is case 2, too)

example 2: $L = \dfrac{\mu_1 + \mu_2}{2} - \dfrac{\mu_3 + \mu_4}{2}$

example 3: $L = \mu_1 - \dfrac{\mu_1 + \mu_2 + \mu_3 + \mu_4}{4}$

Estimation of $\hat{L} = \sum_i^r c_i \bar{Y}_{i.}$

$E(\hat{L}) = L$

$\sigma^2(\hat{L}) = \sigma^2 \sum_i^r \frac{c_i^2}{n_i}$

$s^2(\hat{L}) = MSE \sum_i^r \frac{c_i^2}{n_i}$

Note: $\dfrac{\hat{L} - L}{s(\hat{L})} \sim t_{(n_T - r)}$

Hypothesis Test $H_0: L = 0$

$H_a: L \neq 0$

$t^* = \dfrac{\hat{L}}{s(\hat{L})}$

reject if $|t^*| > t(1 - \alpha/2, n_T - r)$

④ Linear Combination - $\sum_i^r c_i \mu_i$ but still applicable when $\sum c_i \neq 0$

Hypothesis Test $H_0: \sum c_i \mu_i = c$

$H_a: \sum c_i \mu_i \neq c$

$F^* = (t^*)^2$

SAS

```
proc glimmix;
   class x;
   model y=x/s; */(this will give t tests for μ=0)/*;
   lsmestimate x 'mean difference' 0 0 1 -1 /alpha=.01 cl;
                 */(this will give t tests for D=0)/*;
   lsmestimate x 'contrast' .5 .5 -.5 -.5 /alpha=.01 cl;
                 */(this will give t tests for L=0)/*;
   lsmeans x /pdiff adjust = tukey alpha=.01; */(all pairwise differences)*/;
run;
```

<u>Simultaneous Inference</u> - We do this to have intervals we can compare and contrast

3 ways - Tukey, Scheffe, Bonferoni

<u>Tukey</u> Use when the interest is all pairwise differences

$\hat{D} = \bar{Y}_{i\cdot} - Y_{i'\cdot}$

$s^2(\hat{D}) = MSE\left(\frac{1}{n_i} + \frac{1}{n_{i'}}\right)$

$T = \frac{1}{\sqrt{2}} q(1-\alpha, r, n_T - r)$

<u>Confidence Interval</u>: $\hat{D} \pm T s(\hat{D})$

<u>Hypothesis Test</u>: $H_0: \mu_i - \mu_{i'} = 0$

$H_a: \mu_i - \mu_{i'} \neq 0$

$$q^* = \frac{\sqrt{2}\,\hat{D}}{s(\hat{D})}$$

Reject if $|q^*| > q(1-\alpha, r, n_T - r)$

SAS lsmeans x / pdiff adjust=tukey alpha=.01;

· **Scheffé** Interested in contrasts

$H_0: L = \sum c_i \mu_i = 0$

$H_a: L \neq 0$

$$F^* = \frac{\hat{L}^2}{(r-1)s^2(\hat{L})}$$

reject if $F^* > F(1-\alpha, r-1, n_T - r)$

Confidence Int $\hat{L} \pm \sqrt{(r-1)F(1-\alpha; r-1, n_T - r)}\, s(\hat{L})$

Bonferroni Interested in pairwise comparisons, contrasts or linear combinations specified before data analysis.

Confidence Int $\hat{L} \pm t(1-\alpha/2g, n_T - r)s(\hat{L})$

Hyp Test $H_0: L = 0$

$H_a: L \neq 0$

$$t^* = \frac{\hat{L}}{s(\hat{L})}$$

reject if $|t^*| > t(1-\alpha/2g, n_T - r)$

- All pairwise comparisons → Tukey
- Some pairwise comparisons → Bonferroni
- When the number of contrasts = number of factor levels → Bonferroni
- When the number of contrasts >> number of factor levels → Scheffé
- The best method returns the smallest intervals

SAS

```
proc glimmix; class x;
   model y=x/s;
   lsmestimate 'L1'  .5 .5  -.5 -.5,
               'L2'  0  0    1  -1 / adjust= bonf
                                             scheffe  alpha=.01 cl;
run;
```