Chapter 15

Experimental Group - Randomly selected to receive treatment
Control Group - Remaining selected to receive placebo

Comparative Experimental Study - randomization is employed to assign treatments

Treatments - defined by one or more explanatory factors

Comparative Observational Study - Random samples are taken from two or more populations and the observed outcomes are compared

Observational factor - levels of one or more explanatory factors that define populations

Mixed Experimental & Obs study A two factor study consisting of both experimental and observational studies

Factor - an explanatory variable to be studied in an investigation
- experimental factor a factor where the level is assigned at random
- observational factor a factor that is charcteristic of a experimental unit and not controlled by the investigator
- qualitative factor a categorical variable w/ r levels
- quantitative factor described by a numerical quantity

Two-factor study (multi)

Combo	f1	f2
1	1	1
2	1	2
3	2	1
4	2	2
5	3	1
6	3	2

- Say factor 1 has 3 levels and factor 2 has 2 levels
- factor-level combos = 3*2=6
- We can describe these experiments in the crossed table below
- If levels of one or more of the factors are unique to a particular level of another factor we can use nesting.
  - example 3 plants, 3 workers from each plant, w/ the response production over 5 batches
    see table below

Crossed

f2	f1		
	1	2	3
1	X	X	X
2	X	X	X

Nested -

Plant	Operator								
	1	2	3	4	5	6	7	8	9
1	X	X	X	X	X	X	X	X	X
2	X	X	X	X	X	X	X	X	X
3	X	X	X	X	X	X	X	X	X

* Pick a subset of most important factors
* Use a control when general effect of the treatment is unknown
* Randomization - treatments assigned to experimental units at random

Blocking - Used to increase precision
- From one experiment create many
- Say we have $Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$

$$\sigma^2(b_1) = \frac{4s^2}{n}$$

- we can lower this by lowering σ^2, which we can do by blocking on a nuisance or confounding variable say gender giving us:

$$Y_{if} = \beta_0 + \beta_1 X_{i1} + \epsilon_i^f$$
$$Y_{im} = \beta_0 + \beta_1 X_{i1} + \epsilon_i^m$$

Block	Randomization					
1	Treat	Treat	Placebo	...	Treatment	(Male)
2	Placebo	Treat	Placebo	...	Treatment	(Female)

$$Y_{ijk} = \beta_0 + \beta_1 \underset{\text{treatment}}{X_{ij1}} + \beta_2 \underset{\text{gender}}{X_{ij2}} + \epsilon_{ij}$$

β_0 is mean response change due to treatment
β_1 is mean response change due to gender

NOTE: $SSE(\text{Full Model}) = SSTO - SSR(X_1, X_2) = SSTO - [SSR(X_1) + SSR(X_2)]$
$SSE(\text{Reduced}) = SSTO - SSR(X_1)$
$SSE(\text{Full}) = SSE(\text{Reduced}) - SSE(X_2)$
$\therefore$ $SSE(X_2)$ extra sum of squares reduction when adding blocking

Completely Randomized Design (CRD) - Treatments are randomly assigned to experimental units
- most useful when experimental units are homogeneous

$$Y_{ij} = \underset{\text{mean resp.}}{\beta_0} + \beta_1 \underset{I(\text{treat 1})}{X_{ij1}} + \beta_2 \underset{I(\text{treat2})}{X_{ij2}} + \beta_3 \underset{I(\text{treat3})}{X_{ij3}} \cdots + \epsilon_{ij}$$

- We want $\epsilon \sim N(0, \sigma^2) \rightarrow$ then we can use F test

Factorial Experiment : When treatments correspond to the set of all possible combinations of factor levels

$$Y_{ij} = \underbrace{\beta_0}_{\text{mean response}} + \underbrace{\beta_1 X_{ij1}}_{\text{treat 1}} + \underbrace{\beta_2 X_{ij2}}_{\text{treat 2}} + \underbrace{\beta_3 X_{ij1} X_{ij2}}_{\text{interaction}} + \underbrace{\epsilon_{ij}}_{\text{error}}$$

Randomized Complete Block Design - RCBD

- Experimental units are split into homogeneous blocks, then seperate randomization of treatments for each block

Block	EXPER. UNIT			
	1	2	3	4
1	T_1	T_3	T_4	T_2
2	T_3	T_4	T_1	T_2

$$Y_{ij} = \underbrace{\beta_0}_{\text{mean resp}} + \underbrace{\beta_1 X_{ij1}}_{\text{treatment}} + \underbrace{\beta_2 X_{ij2}}_{\text{blocking}} + \underbrace{\epsilon_i}_{\text{error}}$$

- we want $\epsilon \sim N(0, \sigma^2)$ then we can try F test

Nested Design : Blocking within a block

Block 1	Block 2	Exp Unit					
		1	2	3	4	5	6
1	1	X	X				
	2			X	X		
	3					X	X
2	1	X	X				
	2			X	X		
	3					X	X

Repeated Measures Design Each experimental unit receives all the treatments - like a taste test

Exper. Units	treatments		
	1	2	3
1			
2			
3			
4			
⋮			
12			

We can additionally block (Split Plot)

Block	Exp. Units	Treatments		
		1	2	3
1	1 2 ⋮ 6			
2	7 8 9 ⋮ 12			

Incomplete Block Design: When block sizes are smaller than treatments

Block	treatments				
	1	2	3	4	5
1	X	X	X		
2	X	X		X	
3	X	X			X
4	X		X	X	
5	X		X		X
6	X			X	X
7		X	X	X	
8		X	X		X
9		X		X	X
10			X	X	X

← Balanced incomplete Block Design

15.4 and 15.5 omitted