

Chapter 9 Model Building

1. Data collection and preparation
2. Reduction of predictor variables
3. Model refinement and selection
4. Model validation

4 Types of studies

Controlled: • Experimenter - controls levels of explanatory variables
- assigns treatments

Controlled w/ covariates: Same as controlled experiment except here we consider attributes of participants, ie age, education, ad hoc - after the experiment

Confirmatory-Observational: • Based on observational data (not experimental) used to test hunches
• Not controlled as the two above
• control variables: known risk factors
• explanatory variables: hunch factors

Exploratory-Observational: • Looking for predictor variables that are related to a desired response variable

Data prep: • Check for outliers

Reduction of explanatory variables

Controlled: usually not important

controlled w/ covariates: covariates may need reduction

confirmatory: usually not important

Exploratory: **IMPORTANT** - there may be a lot of explanatory variables
some can try to find a 'best subset'

Important variables left out are sometimes called latent vars (S)

9.2 Example

- 1.) Screen for outliers
- 2.) Start with a first order model
 - (a) Does the residual plot indicate
 - i) non-constant variance?
 - ii) curvature?
 - (b) Does the Normality QQ Plot indicate
 - i) departures from normality?
 - (c) Fix issues
 - i) Try box-cox transformation to fix constant-variance
 - (d) Check that predictors are
 - i) linearly associated w/ response
 - ii) not too strongly associated
 - (e) Fix issues
 - i) Add higher order terms
 - ii) collapse variables
 - (f) Refinement
 - i) Can we drop predictors?
 - ii) Is there an adequate subset?

• From any set of $p-1$ predictors there are 2^{p-1} models to choose from

• R_p^2 or SSE_p Criterion $R_p^2 = 1 - \frac{SSE_p}{SSTO}$

• 'Good' when SSE is small and R_p^2 is close to 1

• Used to see when adding variables is no longer useful

• $R_{a,p}^2$ or MSE_p Criterion $R_{a,p}^2 = 1 - \frac{(n-1)}{(n-p)} \frac{SSE_p}{SSTO} = 1 - \frac{MSE_p}{SSTO/(n-1)}$

• 'Good' when $R_{a,p}^2$ is close to 1

Mallow's Cp Criterion Concerned w/ total mean squared error
 ✓ subset regression model

$$C_p = \frac{SSE_p}{MSE(x_1, x_2, \dots, x_{p-1})} - (n - 2p)$$

$$E(C_p) = p \quad \text{when} \quad E(\hat{y}_i) = \mu_i$$

- * Models with little bias will fall near $C_p = p$ line
- * Models with bias will fall considerably above $C_p = p$

AIC_p and SBC_p Criteria

$$AIC_p = n \ln(SSE_p) - n \ln(n) + 2p$$

$$SBC_p = n \ln(SSE_p) - n \ln(n) + \ln(n)p$$

- * Smaller = better for goodness of fit
- * Both penalizes models w/ a lot of predictors

Press Criterion

$$Press_p = \sum (y_i - \hat{y}_{i(i)})^2 = \text{Sum of Squared prediction error}$$

SAS

proc reg;

model y = x₁ x₂ x₃ / selection = cp aic sbc press;

run;

| # of params in model | C(p) | R-sq | AIC | SBC | Variables in model |
|----------------------|------|------|-----|-----|--------------------|
| # | | | | | |
| # | | | | | |
| # | | | | | |
| ⋮ | | | | | |

"Best" Subsets algorithm · Checks different models for Cp AICp SBCp and Pressp to get a small subset of "good" models

SAS

proc reg;

model $y = x_1 x_2 x_3$ / selection = cp best = 3;

run;

* Note: we prefer hierarchical models

"Stepwise Regression Models" · When we have, say 30-40 variables and "Best" would be too cumbersome to run we use this

2 types - Forward Selection
- Backward Elimination

Forward: 1) Simple regression model for each predictor

- $t_k^* = \frac{b_k}{se b_k}$ is calculated

· largest + values yield candidate for addition

2) Regression model with first candidate and each other pred.

· $t_k^* = \frac{b_k}{se b_k}$

· largest + values yield candidate

∴ continue in this fashion

SAS

proc reg;

model $y = x_1 x_2 x_3$ / selection = stepwise slentry = .15 slstay = .20;

run;

backward } can be used instead
or
forward }