Chapter 8: Estimation

Goal of Statistical Inference: Use information in our sample data to make a conclusion about a population of interest.

- In particular, once we have chosen a distribution to be a model for our data, we want to estimate any unknown parameter(s) of that distribution.

Example 1: A company wishes to estimate the mean service time μ for customers.

- The mean, μ, is the ______ ______ (or parameter of interest).

Example 2: A manufacturer wishes to estimate the standard deviation σ of the diameters of a part produced in a factory.

- Here, σ is the ______ ______.

Two Types of Estimate

- A point estimate is a single number.
- An interval estimate is a range of possible values, such as:

- An estimator is a formula for calculating an estimate from data values in a sample.
- The estimate is the actual calculated value.

Judging the Quality of Point Estimators

- We traditionally evaluate estimators based on their values across repeated samples (of the same size) from the same population.
- Since an estimator is a function of the random sample values, it is itself a random variable.
- We want a point estimator to be ________ — to be at or near the target parameter value, on average.
- We also want our point estimator to be ________ — to be consistent in value across repeated samples.
- Good precision ≡ ______ variance.

8.2 Bias and Mean Squared Error of a Point Estimator

- Suppose our target parameter is denoted θ.
- Then let a point estimator of θ be denoted by $\hat{\theta}$.
- Since $\hat{\theta}$ is a r.v., we can find $E(\hat{\theta})$ and $var(\hat{\theta})$.
- We say a point estimator $\hat{\theta}$ is unbiased if:

(if the average value of the estimator equals the target parameter)

- The bias of an estimator $\hat{\theta}$ is:

- While being unbiased is good, unbiasedness alone does not make an estimator desirable.
- An estimator that is far less than θ half the time and equally far above θ the other half of the time is unbiased, but it is not a good estimator of θ.
- We also want our estimator to have low variance.

- The Mean Squared Error (MSE) of an estimator measures a combination of bias and variance:

Note:

- So the MSE of $\hat{\theta}$ equals its variance plus the square of its bias.

Example 1: Let Y be a single observation from a binomial distribution with known n and unknown p. (That is, out of n "trials", Y is our observed number of "successes".) We wish to estimate the true success probability p.

- Let $\hat{p} = \frac{Y}{n}$. Then

- Another estimator of p could be $\hat{p}^* = \frac{Y+1}{n+2}$.

Note:

But note:

Exercise: For a given value of n, plot $MSE(\hat{p}) - MSE(\hat{p}^*)$ against p. See that for values of p near 0.5, then $\hat{p}^*$ has lower MSE than $\hat{p}$.

8.3 Some Common Unbiased Estimators

Situation 1: $Y_1, \ldots, Y_n$ iid with mean μ and variance σ^2.

Then

Situation 2: If $X_1, \ldots, X_n \overset{iid}{\sim}$ Bernoulli (p) and

$Y = \sum_{i=1}^{n} X_i$,

then:

Situation 3:

Two indep. samples:
- $Y_{11}, \ldots, Y_{1n_1}$ iid, mean μ_1 and variance σ_1^2
- $Y_{21}, \ldots, Y_{2n_2}$ iid, mean μ_2 and variance σ_2^2

Then:

Note:

Situation 4:

Two indep. samples:
- $X_{11}, \ldots, X_{1n_1} \overset{iid}{\sim}$ Bernoulli (p_1), let $Y_1 = \sum_i X_{1i}$
- $X_{21}, \ldots, X_{2n_2} \overset{iid}{\sim}$ Bernoulli (p_2), let $Y_2 = \sum_i X_{2i}$

Then:

Note:

Situation 1 again: The sample variance

$$S^2 = \frac{\sum_{i=1}^{n} (Y_i - \overline{Y})^2}{n-1}$$ is an unbiased estimator of σ^2.

Proof:

- Why not use $\hat{\sigma}^{2*} = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \bar{Y})^2$ to estimate σ^2?

Note:

<u>Interesting Fact:</u> Although the sample variance S^2 is an unbiased estimator of σ^2, the sample standard deviation $S = \sqrt{S^2}$ is a <u>biased</u> estimator of the population standard deviation σ.