STAT/MATH 511: Probability

Chapter 2: Probability is the measure of one's belief that a future (random) event will occur.

- A random event is one whose occurrence cannot be predicted with certainty.
- However, we can often understand the long-run relative frequency (proportion) of times the event will occur in a long series of trials.

Examples of random events

1. Toss of a balanced coin
2. Gain/loss of the Dow Jones stock index
3. Mass of a randomly selected particle from a certain population

Types of Probability Definitions

1. Subjective Approach:

2. Relative Frequency Approach:

3. Axiomatic Approach:

Defn: A random experiment is the process by which an observation is made.

Simple Example: We roll a possibly balanced die and record the outcome.

Defn. An outcome (also called a sample point) is the particular result of an experiment.

Example: Consider the outcome "1". What is the probability of this outcome?

Relative Frequency Approach:

Note:

- This relative frequency approach is reasonable when it is possible to conceive of many identical repetitions of the experiment.

Note: If die is balanced, $P(1) =$

- Suppose out of 50 tosses, we obtained no "1"s. Conclusion?
- Is it impossible to get no 1's in 50 tosses of a balanced die?
- Is this unlikely?
- Our conclusion is an inference based on a probability.

2.3 Set Notation

- We use capital letters to denote sets of points (points are typically outcomes in our case).
- Example 1:

- Special Sets: $S =$

$\emptyset =$

Defn: A is a subset of B if every point in A is also in B.

Example 1(a): B =

Then:

A Venn Diagram can graphically display simple sets.

Example 1:

If C = then:

Defn: The union of A and B (denoted $A \cup B$) is the set of all points in A or in B or in both.

Defn: The intersection of A and B (denoted $A \cap B$) is the set of all points in A and in B (simultaneously).

Example 1(b): In previous example,

$A \cap B =$

Example 1(c): In previous example,

$B \cap C =$

$B \cup C =$

Defn: If A is a subset of S, the complement of A (denoted $\overline{A}$) is the set of points that are in S but not in A.

Venn Diagram:

Note:

Defn: Sets A and B are mutually exclusive (or disjoint) if $A \cap B = \emptyset$.

- Mutually exclusive sets have no points in common.

Venn Diagram:

Die-rolling example: S =

Let A = {roll number less than 3}

Let B = {roll odd number}

Let C = {roll even number}

$A \cap B =$

$A \cap C =$

$A \cup C =$

$\overline{A} =$

-Which are mutually exclusive?

A and B? A and C? B and C?

Important Laws

Distributive Laws:

DeMorgan's Laws:

Venn Diagrams: