STAT 509 – Section 4.1 – Estimation

Parameter: A numerical characteristic of a population.

Examples:

Statistic: A quantity that we can calculate from sample data that summarizes a characteristic of that sample.

Examples:

Point Estimator: A statistic which is a single number meant to estimate a parameter.

It would be nice if the average value of the estimator (over repeated sampling) equaled the target parameter.

An estimator is called unbiased if the mean of its sampling distribution is equal to the parameter being estimated.

Examples:

Another nice property of an estimator: we want it to be as *precise* as possible.

The standard deviation of a statistic's sampling distribution is called the <u>standard error</u> of the statistic.

The standard error of the sample mean $\overline{\mathbf{Y}}$ is $\sigma / \sqrt{n}$.

Note: As the sample size gets larger, the spread of the sampling distribution gets smaller.

When the sample size is large, the sample mean <u>varies less</u> across samples.

<u>Evaluating an estimator</u>:

(1) Is it unbiased?
(2) Does it have a small standard error?

<u>Interval Estimates</u>

• With a point estimate, we used a single number to estimate a parameter.
• We can also use a <u>set of numbers</u> to serve as "reasonable" estimates for the parameter.

Example: Assume we have a sample of size *n* from a normally distributed population.

We know: $T = \dfrac{\bar{Y} - \mu}{s/\sqrt{n}}$ **has a t-distribution with *n* – 1 degrees of freedom.**
(Exactly true when data are normal, approximately true when data non-normal but *n* is large.)

So: 1 – α = $P\left(-t_{n-1,\alpha/2} \le \dfrac{\bar{Y} - \mu}{s/\sqrt{n}} \le t_{n-1,\alpha/2}\right)$

=

where $t_{n-1,\,\alpha/2}$ **= the t-value with α/2 area to the right (can be found from Table 2)**

This formula
is called a "confidence interval" for μ.

<u>Confidence Interval</u>: An interval (along with a level of confidence) used to estimate a parameter.

• Values in the interval are considered "reasonable" values for the parameter.

<u>Confidence level</u>: The percentage of all CIs (if we took many samples, each time computing the CI) that contain the true parameter.

Example: If $\overline{Y}$ is normally distributed, then in 95% of samples, the interval

will contain μ.

<u>Example 1</u>: We want to estimate the mean Rockwell hardness μ of the heads of shearing pins. We take a sample of 12 pins. Sample mean $\overline{Y}$ = 48.5, s = 1.5. Assume the measurements follow a normal distribution. What is a 95% CI for μ?

<u>Question</u>: What does 95% confidence <u>mean</u> here, exactly?

• If we took many samples and computed many 95% CIs, then about 95% of them would contain μ.

The fact that contains μ "with 95% confidence" implies the <u>method</u> used would capture μ 95% of the time, if we did this over many samples.

Picture:

<u>Note</u>: The endpoints of the CI are <u>statistics</u>, calculated from sample data. (The <u>endpoints</u> are random, not the parameter!)

<u>A WRONG statement</u>: "There is .95 probability that μ is between 47.55 and 49.45." Wrong! μ is not random – μ doesn't change from sample to sample. It's either between 47.55 and 49.45 or it's not.

Interpreting a 95% Confidence Interval: TRUE or FALSE?

(1) 95% of all pins have hardness between 47.55 and 49.45.

(2) 95% of samples have $\overline{Y}$ between 47.55 and 49.45.

(3) 95% of samples will produce intervals that contain μ.

(4) 95% of the time, μ is between 47.55 and 49.45.

(5) The probability that μ falls within a 95% CI is 0.95.

(6) The probability that μ falls between 47.55 and 49.45 is 0.95.

Level of Confidence

Recall example: 95% CI for μ was (47.55, 49.45).
• For a 90% CI, we use $t_{.05}$ (11 d.f.) = 1.796.
• For a 99% CI, we use $t_{.005}$ (11 d.f.) = 3.106.

90% CI:

99% CI:

Note tradeoff: If we want a higher confidence level, then the interval gets wider (less precise).

Sample Size Determination

Suppose we actually knew (or could guess) the true value of σ. What would be a CI formula for μ?

Note that the margin of error *B* of this CI for μ is:

Note: When the sample size n is bigger, the CI is narrower (more precise).

We often want to determine what sample size we need to achieve a pre-specified margin of error and level of confidence. Solving for n:

Note: Always round n <u>up</u> to the next largest integer.

Example 2: How many light bulbs do we need for a failure time study? We want a 95% CI for mean lifetime, with a margin of error of 10 hours. We believe that σ = 40 hours.