

## Test 1 Formulas

If  $Y_1, \dots, Y_n$  are iid continuous r.v.'s  
with pdf  $f(y)$  and cdf  $F(y)$  :

$$f_{Y_{(k)}}(y) = \frac{n!}{(k-1)!(n-k)!} [F(y)]^{k-1} f(y) [1-F(y)]^{n-k}$$

with appropriately defined support.

For  $Y_{(j)}$  and  $Y_{(k)}$ ,  $j < k$  :

$$f_{Y_{(j)}, Y_{(k)}}(y_j, y_k) = \frac{n!}{(j-1)!(k-1-j)!(n-k)!} [F(y_j)]^{j-1} f(y_j) [F(y_k) - F(y_j)]^{k-1-j} f(y_k) [1-F(y_k)]^{n-k}$$

with appropriately defined support.

## Discrete Distributions

Distribution	Probability Function	Mean	Variance	Moment-Generating Function
Binomial	$p(y) = \binom{n}{y} p^y (1-p)^{n-y};$ $y = 0, 1, \dots, n$	$np$	$np(1-p)$	$[pe^t + (1-p)]^n$
Geometric	$p(y) = p(1-p)^{y-1};$ $y = 1, 2, \dots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-(1-p)e^t}$
Hypergeometric	$p(y) = \frac{\binom{r}{y} \binom{N-r}{n-y}}{\binom{N}{n}};$ $y = 0, 1, \dots, n \text{ if } n \leq r,$ $y = 0, 1, \dots, r \text{ if } n > r$	$\frac{nr}{N}$	$n \left(\frac{r}{N}\right) \left(\frac{N-r}{N}\right) \left(\frac{N-n}{N-1}\right)$	
Poisson	$p(y) = \frac{\lambda^y e^{-\lambda}}{y!};$ $y = 0, 1, 2, \dots$	$\lambda$	$\lambda$	$\exp[\lambda(e^t - 1)]$
Negative binomial	$p(y) = \binom{y-1}{r-1} p^r (1-p)^{y-r};$ $y = r, r+1, \dots$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\left[ \frac{pe^t}{1-(1-p)e^t} \right]^r$

Distribution	Probability Function	Mean	Variance	Function
Uniform	$f(y) = \frac{1}{\theta_2 - \theta_1}; \theta_1 \leq y \leq \theta_2$	$\frac{\theta_1 + \theta_2}{2}$	$\frac{(\theta_2 - \theta_1)^2}{12}$	$\frac{e^{\theta_2} - e^{\theta_1}}{t(\theta_2 - \theta_1)}$
Normal	$f(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\left(\frac{1}{2\sigma^2}\right)(y - \mu)^2\right]$ $-\infty < y < +\infty$	$\mu$	$\sigma^2$	$\exp\left(\mu t + \frac{t^2\sigma^2}{2}\right)$
Exponential	$f(y) = \frac{1}{\beta} e^{-y/\beta}; \beta > 0$ $0 < y < \infty$	$\beta$	$\beta^2$	$(1 - \beta t)^{-1}$
Gamma	$f(y) = \left[\frac{1}{\Gamma(\alpha)\beta^\alpha}\right] y^{\alpha-1} e^{-y/\beta};$ $0 < y < \infty$	$\alpha\beta$	$\alpha\beta^2$	$(1 - \beta t)^{-\alpha}$
Chi-square	$f(y) = \frac{(y)^{(v/2)-1} e^{-y/2}}{2^{v/2} \Gamma(v/2)};$ $y^2 > 0$	$v$	$2v$	$(1 - 2t)^{-v/2}$
Beta	$f(y) = \left[\frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)}\right] y^{\alpha-1} (1 - y)^{\beta-1};$ $0 < y < 1$	$\frac{\alpha}{\alpha + \beta}$	$\frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$	does not exist in closed form