

## Two-level designs

In this exercise, we will focus on the analysis of an unreplicated full factorial two-level design, typically referred to as a  $2^k$  design— $k$  factors, all crossed, with two levels each. I had discussed replicated designs as well, but unreplicated designs have their own particular methods for inference that are quite different from methods used for replicated factorial completely randomized designs (CRD's), and I would like to reinforce some of these differences with this exercise.

For our data set, we will use a former statistics student's (Carla Padgett) class project under Prof. Don Edwards, in which the loudness of a violin's note (measured in decibels) was modelled as a function of 4 2-level factors. This experiment was actually replicated (11 times!), but we will study the averages here, and treat it as an unreplicated experiment.

Table 1. Factors and Factor Levels

Factor	Factor Label	-1	1
A	Bow Speed	Low	High
B	Bow Angle	Low	High
C	Bow Placement	Near	Far
D	Pressure	Low	High

Table 2. Responses and Runs

Decibels (Y)	A	B	C	D
69.3	-1	-1	-1	-1
75.3	-1	-1	-1	1
75.9	-1	-1	1	-1
79.3	-1	-1	1	1
67.4	-1	1	-1	-1
74.9	-1	1	-1	1
74.4	-1	1	1	-1
78.8	-1	1	1	1
73.4	1	-1	-1	-1
77.5	1	-1	-1	1
78.8	1	-1	1	-1
81.6	1	-1	1	1
72.3	1	1	-1	-1
78.8	1	1	-1	1
78.1	1	1	1	-1
81.5	1	1	1	1

Note that Table 2 has the data entered in the “wrong” order, with the levels of Factor A varying the slowest, while the levels of Factor D vary the quickest. When creating the design in Minitab (see “Some notes on factorial designs in Minitab” on the course website for a primer), we could actually modify the way in which we specify the design using `Stat>DOE>Factorial>Create Customized Design`. It’s easier, however, just to reverse the labels of the factors when using `Stat>DOE>Factorial>Create Factorial Design`, so that we enter our Factor D as Minitab’s Factor A, etc. We do this by selecting the Factor button in the `Stat>DOE>Factorial>Create Factorial Design` interface (after having selected a 4-factor full factorial design), and then entering information on labels and levels for our factors in reverse order. After we have created the factorial design, we can then enter the response variable in the Minitab worksheet in its original order. Inspect the Minitab worksheet to make sure that both your labeling and run order are correct; the Minitab worksheet should look something like this:

Table 3. Minitab Worksheet

StdOrder	RunOrder	CenterPt	Blocks	Pressure	Bow Placement	Bow Angle	Bow Speed	Decibels
1	1	1	1	Low	Near	Low	Low	69.3
2	2	1	1	High	Near	Low	Low	73.4
3	3	1	1	Low	Far	Low	Low	67.4
4	4	1	1	High	Far	Low	Low	72.3
5	5	1	1	Low	Near	High	Low	75.9
6	6	1	1	High	Near	High	Low	78.8
7	7	1	1	Low	Far	High	Low	74.4
8	8	1	1	High	Far	High	Low	78.1
9	9	1	1	Low	Near	Low	High	75.3
10	10	1	1	High	Near	Low	High	77.5
11	11	1	1	Low	Far	Low	High	74.9
12	12	1	1	High	Far	Low	High	78.7
13	13	1	1	Low	Near	High	High	79.3
14	14	1	1	High	Near	High	High	81.6
15	15	1	1	Low	Far	High	High	78.8
16	16	1	1	High	Far	High	High	81.5

With the responses entered (remember to add a name to this column, e.g., Decibels), we can now analyze the design. Again, refer to the website for the basics on analysis; be sure to select “Effects Plot” under Graphs. I usually choose the Normal Effects Plot, but for this demonstration, I also selected the Half Normal Effects Plot. You should get output in the Session window that looks like this:

Factorial Fit: Decibels versus Pressure, Bow Placement, ...

Estimated Effects and Coefficients for Decibels (coded units)

Term	Effect	Coef
Constant		76.0750
Pressure	3.3250	1.6625
Bow Placement	-0.6250	-0.3125
Bow Angle	4.9500	2.4750
Bow Speed	4.7500	2.3750
Pressure*Bow Placement	0.4500	0.2250
Pressure*Bow Angle	-0.4250	-0.2125
Pressure*Bow Speed	-0.5750	-0.2875
Bow Placement*Bow Angle	-0.0750	-0.0375
Bow Placement*Bow Speed	0.6750	0.3375
Bow Angle*Bow Speed	-1.2500	-0.6250
Pressure*Bow Placement*Bow Angle	-0.1500	-0.0750
Pressure*Bow Placement*Bow Speed	0.0500	0.0250
Pressure*Bow Angle*Bow Speed	0.1750	0.0875
Bow Placement*Bow Angle*Bow Speed	-0.2750	-0.1375
Pressure*Bow Placement*Bow Angle* Bow Speed	-0.1500	-0.0750

S = \* PRESS = \*

Analysis of Variance for Decibels (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	4	234.045	234.045	58.5112	*	*
2-Way Interactions	6	10.950	10.950	1.8250	*	*
3-Way Interactions	4	0.525	0.525	0.1312	*	*
4-Way Interactions	1	0.090	0.090	0.0900	*	*
Residual Error	0	*	*	*		
Total	15	245.610				

Some of the information in the Session Window is not that interesting, simply because we have a saturated unreplicated design. With no degrees of freedom for errors, Minitab will not produce individual standard errors and t-tests for each effect, and

the ANOVA table will not include F tests for the referenced effects. It's a peculiar choice of Minitab to decompose the ANOVA table by main effects, two-way effects, etc; I would prefer that Minitab go ahead and list the F statistic and test for each effect separately, but it doesn't. Of course, this information is available from the t tests (when error df is not 0), but I don't necessarily find Minitab's grouping of effects by their order to be particularly useful. Note that the coefficient and effect estimates follow the pattern I discussed in class: the coefficients are always half the size of the effects and they share the same sign.

The effects plots shows that Pressure, Bow Angle and Bow Speed are significant at the  $\alpha = .05$  level using Lenth's test. Sometimes I feel that Lenth's test has too strong an influence on students' decision-making; it's hard to ignore those bright red, labelled squares.

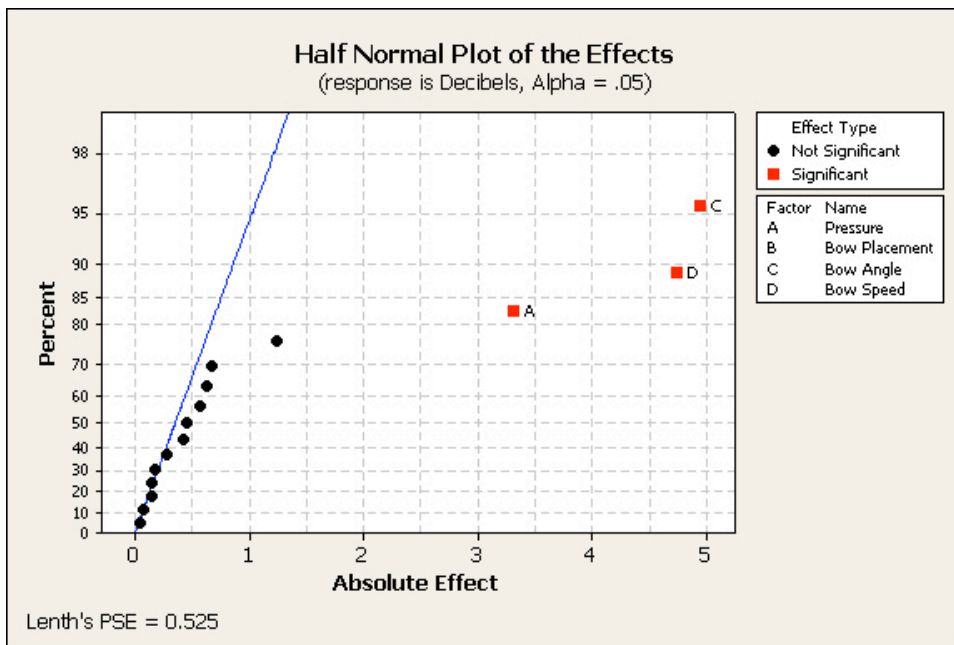


Figure 1: Half Normal Plot of Decibel Effects

An inspection of either the normal plot or the half normal plot suggests that one of the unlabelled terms could be marginally significant. If we refer back to the Session window, we can see that this was the Bow Angle by Bow Speed interaction. Since these are both significant main effects, it may be interesting to construct an interaction plot. Note that the interaction effect is negative while the two main effects are positive; this means that either effect at its high level will act as a suppressor; e.g., the main effect of Bow Speed at the high level of Bow Angle will be *smaller* than

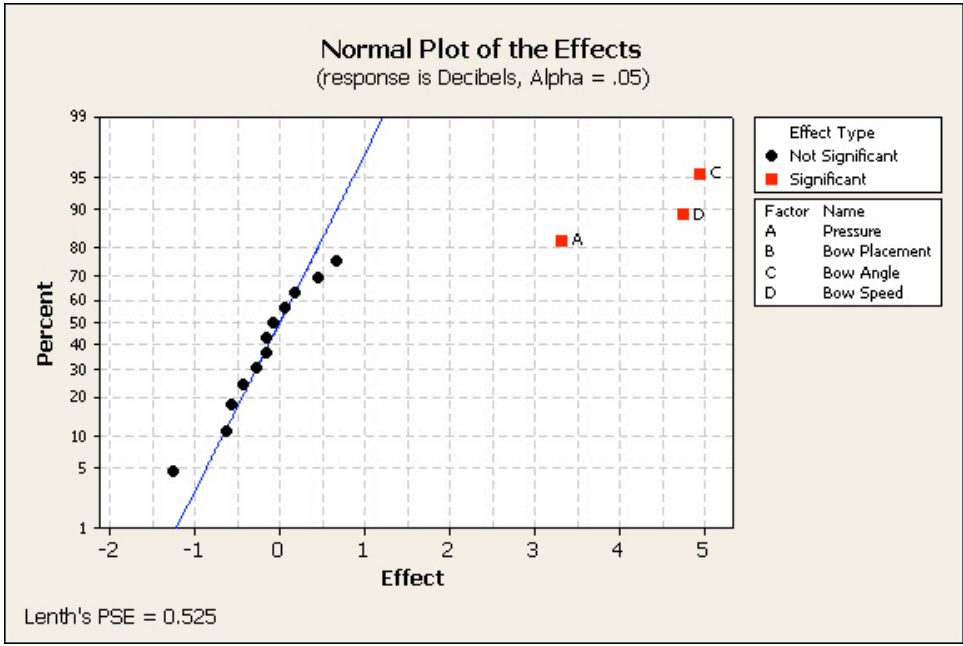


Figure 2: Normal Plot of Decibel Effects

the main effect of Bow Speed at the low level of Bow Angle. We can construct an interaction plot (using ANOVA>Interaction Plot—see the website again) to confirm this result. The interaction plot confirms that the effect is marginal; in general, the high bow angle produces a louder sound than the low bow angle, but this effect is less pronounced for a high bow speed.

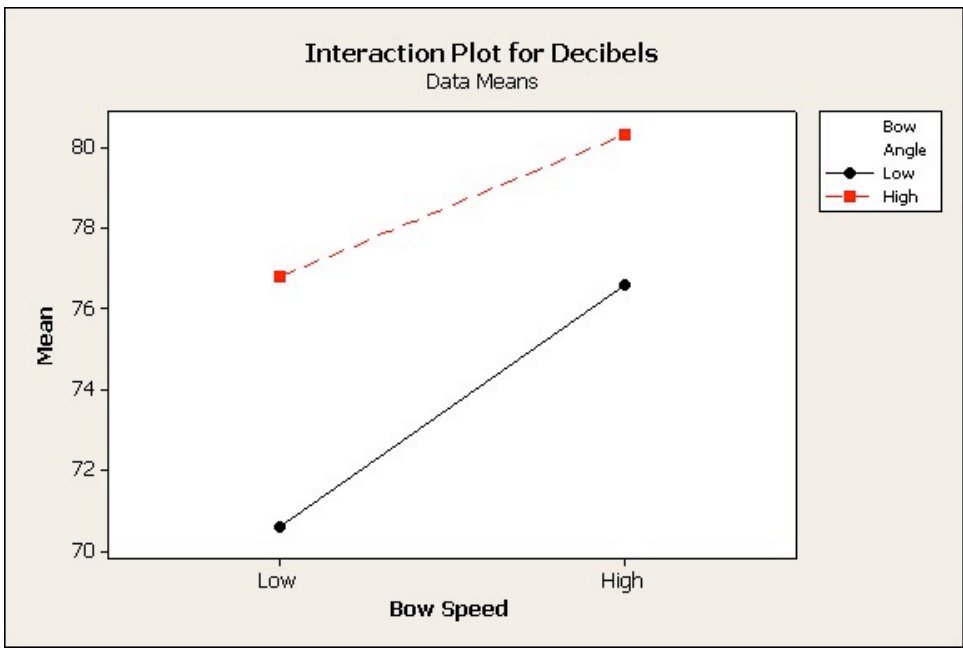


Figure 3: Interaction of Bow Angle and Bow Speed