1. (a) $\mu_1' = EY = \theta$, $m_1' = \bar{Y}$, $m_1' = \mu_1' \Rightarrow \hat{\theta}_{MOM} = \bar{Y}$

(b) $L(\theta|\underline{Y}) = \prod_{i=1}^{n}\left(\frac{e^{-\theta}\theta^{y_i}}{y_i!}\right) = \frac{e^{-n\theta}\theta^{\sum y_i}}{\prod(y_i!)}$

$l(\theta|\underline{Y}) = \sum y_i \ln\theta - n\theta - \ln\prod(y_i!)$

$\frac{\partial l}{\partial\theta} = \frac{\sum y_i}{\theta} - n = 0$, $\frac{\partial l}{\partial\theta^2} = -\frac{\sum y_i}{\theta^2} < 0 \Rightarrow \hat{\theta}_{MLE} = \bar{Y}$

(c) $T = \bar{Y}$ or $\sum y_i$

(d) $\hat{\theta}_{MVUE} = \bar{Y}$

2 (a) CLT $\Rightarrow \frac{\bar{Y}-\theta}{\sqrt{\frac{\theta}{n}}} \xrightarrow{d} N(0,1)$

WLLN $\Rightarrow \bar{Y} \xrightarrow{P} \theta \Rightarrow \frac{\bar{Y}}{\theta} \xrightarrow{P} 1 \Rightarrow \sqrt{\frac{\bar{Y}}{\theta}} \xrightarrow{P} 1$

Slutsky $\Rightarrow W_n = \frac{\bar{Y}-\theta}{\sqrt{\bar{Y}/n}} \xrightarrow{d} N(0,1)$

(b) $\bar{Y} \pm 1.96\sqrt{\frac{\bar{Y}}{n}}$

3 $\mu_1' = E(Y) = \frac{a+b}{2}$, $Var(Y) = \frac{(b-a)^2}{12}$

$\mu_2' = E(Y^2) = (EY)^2 + Var(Y) = \frac{(a+b)^2}{4} + \frac{(b-a)^2}{12} = \frac{1}{3}(a^2+b^2+ab)$

$m_1' = \bar{Y}$

$m_2' = \frac{1}{n}\sum y_i^2$

$m_1' = \mu_1' \Rightarrow a+b = 2\bar{Y}$ ①

$m_2' = \mu_2' \Rightarrow \frac{(a+b)^2}{4} + \frac{(b-a)^2}{12} = m_2' \Rightarrow b-a = \pm 2\sqrt{3(m_2'-\bar{Y}^2)}$ ②

$\hat{a} = \bar{Y} - \sqrt{3(m_2'-\bar{Y}^2)}$

$\hat{b} = \bar{Y} + \sqrt{3(m_2'-\bar{Y}^2)}$

4 $f_Y(y) = \frac{1}{\Gamma(\alpha_0)\beta^{\alpha_0}} y^{\alpha_0-1} e^{-y/\beta}$

$L(\theta|\beta) = \left[\frac{1}{\Gamma(\alpha_0)\beta^{\alpha_0}}\right]^n (\prod y_i)^{\alpha_0-1} e^{-\sum y_i/\beta}$

$l(\theta|\beta) = -n\ln\Gamma(\alpha_0) - n\alpha_0\ln\beta + (\alpha_0-1)\ln(\prod y_i) - \frac{1}{\beta}\sum y_i$

$\frac{\partial l}{\partial\beta} = -n\frac{\alpha_0}{\beta} + \frac{\sum y_i}{\beta^2} = 0 \Rightarrow \hat{\beta} = \frac{\bar{Y}}{\alpha_0}$

$\left.\frac{\partial l}{\partial\beta}\right|_{\beta=\bar{Y}/\alpha_0} < 0$

5 $2\bar{X} + \bar{Y} \pm Z_{\alpha/2}\sigma\sqrt{\frac{7}{n}}$, check 8.93 (a) for details