1. $F_{Y_{(1)}}(y) = P[Y_{(1)} \le y] = 1 - P[Y_{(1)} > y] = 1 - \prod P(Y_i > y) = 1 - [P(Y<y)]^n = 1 - [1 - F_Y(y)]^n$

$= 1 - (e^{-y/\beta})^n = 1 - e^{-\frac{y}{\beta/n}}$

$Y_{(1)} \sim exp(\frac{\beta}{n})$

2. $M_S(t) = [M_Y(t)]^n = (\frac{1}{1-\beta t})^{\alpha n} \Rightarrow S \sim gamma(n\alpha, \beta)$

$M_U(t) = E(e^{tU}) = E(e^{t2S/\beta}) = M_S(\frac{2t}{\beta}) = (\frac{1}{1-\beta\frac{2t}{\beta}})^{\alpha n} = (\frac{1}{1-2t})^{\alpha n} \Rightarrow U \sim \chi^2(2n\alpha)$

3. Let X be the number of voters at precinct 1 who votes for candidate A.

CLT $\Rightarrow \bar{X} \sim AN(0.5, 0.05^2)$

$P(\bar{X} \ge 0.55) = P(\frac{X-0.5}{0.05} \ge \frac{0.55-0.5}{0.05}) = P(Z \ge 1) = 0.16$

4. $\begin{cases} U_1 = \frac{Y_1}{Y_1+Y_2} = h_1(Y_1, Y_2) \\ U_2 = Y_1+Y_2 = h_2(Y_1, Y_2) \end{cases} \Rightarrow \begin{array}{l} Y_1 = U_1U_2 = h_1^{-1}(U_1, U_2) \\ Y_2 = U_2(1-U_1) = h_2^{-1}(U_1, U_2) \end{array} \quad \begin{cases} 0 < U_1 < 1 \\ 0 < U_2 < \infty \end{cases}$

$J = \begin{vmatrix} \frac{\partial h_1^{-1}}{\partial u_1} & \frac{\partial h_1^{-1}}{\partial h_2} \\ \frac{\partial h_2^{-1}}{\partial u_1} & \frac{\partial h_2^{-1}}{\partial u_2} \end{vmatrix} = \begin{vmatrix} u_2 & u_1 \\ -u_2 & 1-u_1 \end{vmatrix} = u_2$

$f_{Y_1,Y_2}(y_1, y_2) = f_{Y_1}(y_1) f_{Y_2}(y_2) = \frac{1}{\beta^2} exp\{\frac{-(y_1+y_2)}{\beta}\}$

$f_{U_1,U_2}(u_1, u_2) = f_{Y_1,Y_2}[h_1^{-1}(u_1,u_2), h_2^{-1}(u_1,u_2)]\ |J|$

$= \frac{1}{\beta^2} exp\{\frac{u_1u_2 + u_2 - u_1u_2}{-\beta}\}\ u_2$

$= \frac{1}{\beta^2} u_2 e^{-u_2/\beta}$

$U_1 \sim uniform(0,1)$, $f_{U_1}(u_1) = 1$

$U_2 \sim gamma(2, \beta)$, $f_{U_2}(u_2) = \frac{1}{\beta^2} u_2 e^{-u_2/\beta}$

5. CLT $\Rightarrow \frac{\bar{Y} - \alpha_0\beta}{\sqrt{\frac{\alpha_0\beta^2}{n}}} \sim AN(0,1)$

CI is $-z_{\frac{\alpha}{2}} < \frac{\bar{Y} - \alpha_0\beta}{\sqrt{\alpha_0\beta^2/n}} < z_{\alpha/2}$

$-z_{\frac{\alpha}{2}} < \frac{\bar{Y}/\beta - \alpha_0}{\sqrt{\alpha_0/n}} < z_{\frac{\alpha}{2}}$

$\alpha_0 - z_{\alpha/2}\sqrt{\frac{\alpha_0}{n}} < \frac{\bar{Y}}{\beta} < \alpha_0 - z_{\alpha/2}\sqrt{\frac{\alpha_0}{n}}$

$\frac{1}{\alpha_0 + z_{\alpha/2}\sqrt{\frac{\alpha_0}{n}}} < \frac{\beta}{\bar{Y}} < \frac{1}{\alpha_0 - z_{\alpha/2}\sqrt{\frac{\alpha_0}{n}}}$

$\frac{\bar{Y}}{\alpha_0 + z_{\alpha/2}\sqrt{\frac{\alpha_0}{n}}} < \beta < \frac{\bar{Y}}{\alpha_0 - z_{\alpha/2}\sqrt{\frac{\alpha_0}{n}}}$