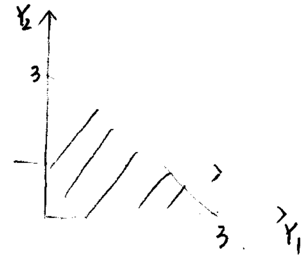
1(a). $f_{Y_1,Y_2}(y_1,y_2) = e^{-y_1} e^{-y_2}, \quad y_1>0,\ y_2>0 \Rightarrow Y_1 \perp Y_2$

$Y_1, Y_2 \sim exp(1)$. $\quad F(y) = 1-e^{-y}$

$P(Y_1<1, Y_2>5) = P(Y_1<1)\cdot P(Y_2>5) = F_{Y_1}(1)\,[1-F_{Y_2}(5)]$

$= (1-e^{-1})\,e^{-5} = \frac{1}{e^5} - \frac{1}{e^6} = 4.259\times 10^{-3}$

(b) $P(Y_1+Y_2<3) = \int_{y_2=0}^{3}\int_{y_1=0}^{3-y_2} e^{-y_1}e^{-y_2}\,dy_1\,dy_2$



$= \int_{y_2=0}^3 e^{-y_2}\left[e^{-y_1}\right]_{3-y_2}^{0} dy_2 = \int_0^3 e^{-y_2}(1-e^{y_2-3})\,dy_2$

$= \int_0^3 (e^{-y_2}-e^{-3})\,dy_2 = e^{-y_2}\Big|_3^0 - e^{-3}y_2\Big|_0^3$

$= 1-e^{-3}-e^{-3}\times 3 = 1-\frac{4}{e^3} = 0.8009$, Note $Y_1+Y_2 \sim gamma(2,1)$, not $exp(2)$

(c) $f_{Y_1|Y_2}(y_1|y_2) = f_{Y_1}(y_1) = e^{-y_1}, \quad y_1>0$

2(a) $M_Y(t) = \left(\frac{1}{1-\beta t}\right)^\alpha$. Note page 89

(b) $E(Y^a) = \int_0^\infty y^a \frac{1}{\Gamma(\alpha)\beta^\alpha} y^{\alpha-1} e^{-\frac{y}{\beta}}\,dy = \int_0^\infty \frac{1}{\Gamma(a+\alpha)\beta^{a+\alpha}} y^{a+\alpha-1}e^{-\frac{y}{\beta}}\,dy \cdot \frac{\Gamma(a+\alpha)\beta^{a+\alpha}}{\Gamma(\alpha)\beta^\alpha}$

$= \frac{\Gamma(a+\alpha)\beta^a}{\Gamma(\alpha)}$

3(a) $F(y) = 1-e^{-y/\beta} = 1-e^{-y/2.4}$

$P(Y>3) = 1-F(3) = e^{-3/2.4} = 0.2865$

(b) $P(2<Y<3) = F(3)-F(2) = e^{-2/2.4} - e^{-3/2.4} = 0.4346 - 0.2865 = 0.1481$

4 $E(Y) = \int_{-\infty}^\infty \frac{y}{2\sigma} e^{-\frac{|y-\mu|}{\sigma}}\,dy = \int_\mu^\infty \frac{y}{2\sigma}\exp\{-\frac{(y-\mu)}{\sigma}\}\,dy + \int_{-\infty}^\mu \frac{y}{2\sigma}\exp\{\frac{y-\mu}{\sigma}\}\,dy$

$\int_\mu^\infty \frac{y}{2\sigma}\exp\{-\frac{(y-\mu)}{\sigma}\}\,dy = \int_\mu^\infty \frac{-y}{2}\, d\left[e^{-\frac{(y-\mu)}{\sigma}}\right] = \frac{y\exp\{-\frac{(y-\mu)}{\sigma}\}}{2}\Big|_\infty^\mu - \int_\infty^\mu e^{-\frac{(y-\mu)}{\sigma}}\frac{1}{2}\,dy$

$= \frac{\mu}{2} - \frac{1}{2}\times(-\sigma)\times\int_\infty^\mu d\left[e^{-\frac{(y-\mu)}{\sigma}}\right] = \frac{\mu}{2}+\frac{\sigma}{2}e^{-\frac{(y-\mu)}{\sigma}}\Big|_\infty^\mu = \frac{\mu}{2}+\frac{\sigma}{2}(1-0) = \frac{\mu}{2}+\frac{\sigma}{2}$

$\int_{-\infty}^\mu \frac{y}{2\sigma}\exp\{\frac{y-\mu}{\sigma}\}\,dy = \int_{-\infty}^\mu \frac{y}{2}\,d(e^{\frac{y-\mu}{\sigma}}) = \frac{y e^{\frac{y-\mu}{\sigma}}}{2}\Big|_{-\infty}^\mu - \int_{-\infty}^\mu e^{\frac{y-\mu}{\sigma}}\frac{1}{2}\,dy$

$= \frac{\mu}{2} - \frac{1}{2}\sigma\int_{-\infty}^\mu d\,e^{(\frac{y-\mu}{\sigma})} = \frac{\mu}{2} - \frac{\sigma}{2} e^{\frac{y-\mu}{\sigma}}\Big|_{-\infty}^\mu = \frac{\mu}{2}-\frac{\sigma}{2}(1-0) = \frac{\mu}{2}-\frac{\sigma}{2}$

$E(Y)=\mu$, Note that $f_Y(y)$ is symmetric about $y=\mu$.

5. (a) $f_{Y|\lambda}(y|\lambda) = \frac{e^{-\lambda}\lambda^y}{y!}$

$f_\lambda(\lambda) = e^{-\lambda}$

$f_{Y,\lambda}(y,\lambda) = f_{Y|\lambda}(y|\lambda) f_\lambda(\lambda) = \frac{e^{-2\lambda}\lambda^y}{y!}$

$f_Y(y) = \int_0^\infty f_{Y,\lambda}(y,\lambda)\, d\lambda = \frac{1}{y!}\int_0^\infty e^{-2\lambda}\lambda^y\, d\lambda$

$= \frac{1}{y!}\int_0^\infty \frac{1}{\Gamma(y+1)\left(\frac{1}{2}\right)^{y+1}} \lambda^{(y+1)-1} e^{-\lambda/\frac{1}{2}}\, d\lambda \quad \frac{\Gamma(y+1)}{2^{y+1}}$

$= \frac{\Gamma(y+1)}{y!\, 2^{y+1}} = \frac{y!}{y!\, 2^{y+1}} = \left(\frac{1}{2}\right)^{y+1}$

(b) $E(Y) = E[E(Y|\lambda)] = E(\lambda) = 1$