

This exam contains 5 questions; each question is worth 10 points. Print your name at the top of this page in the upper right hand corner. You have 2 hours to complete this exam. GOOD LUCK!!

1. Five identical bowls are labeled 1, 2, 3, 4, and 5. Bowl  $i$  contains  $i$  white and  $5 - i$  black balls, with  $i = 1, 2, 3, 4, 5$ . A bowl is randomly selected and two balls are randomly selected (without replacement) from the contents of the bowl.

- What is the probability that both balls selected are white?
- Given that both balls selected are white, what is the probability that bowl 3 was selected?

2. Suppose that  $Y$  has a poisson distribution with parameter  $\lambda$  so that the pmf of  $Y$  is

$$P(Y = y) = \begin{cases} \frac{\lambda^y e^{-\lambda}}{y!}, & y = 0, 1, 2, \dots \\ 0, & \text{otherwise.} \end{cases}$$

- Prove that  $\sum_{y=0}^{\infty} P(Y = y) = 1$
- Find the moment-generating function for  $Y$ .
- Prove that the mean of  $Y$  is  $\lambda$ .
- Prove that the variance of  $Y$  is  $\lambda$ .

3. If  $Y$  has a geometric distribution with success probability  $p$ ,

- Find  $P(Y = \text{an even integer})$
- Define  $X = Y - 1$ . If  $Y$  is interpreted as the number of the trial on which the first success occurs, then  $X$  can be interpreted as the number of failures before the first success. Find the pmf of  $X$ , the mean of  $X$ , and the variance of  $X$ .

4. A standard deck of cards contains 52 cards in four suits (clubs, diamonds, hearts, and spades) and thirteen ranks running from two to ten, jack, queen, king, and ace. Five cards are dealt at random without replacement from a standard deck of 52 cards. What is the probability that

- the hand contains all spades?
- two cards are kings, two cards are queens and one card is jack.
- the hand contains all four aces if it is known that it contains at least three aces.

5. A random variable  $Y$  has a logarithmic series distribution with parameter  $p$  if

$$P(Y = y) = \begin{cases} \frac{-(1-p)^y}{y \cdot \ln(p)}, & y = 1, 2, \dots, 0 < p < 1 \\ 0, & \text{otherwise.} \end{cases}$$

- Prove that  $\sum_{y=1}^{\infty} P(Y = y) = 1$
- Find that the mean of  $Y$ , the variance of  $Y$ .